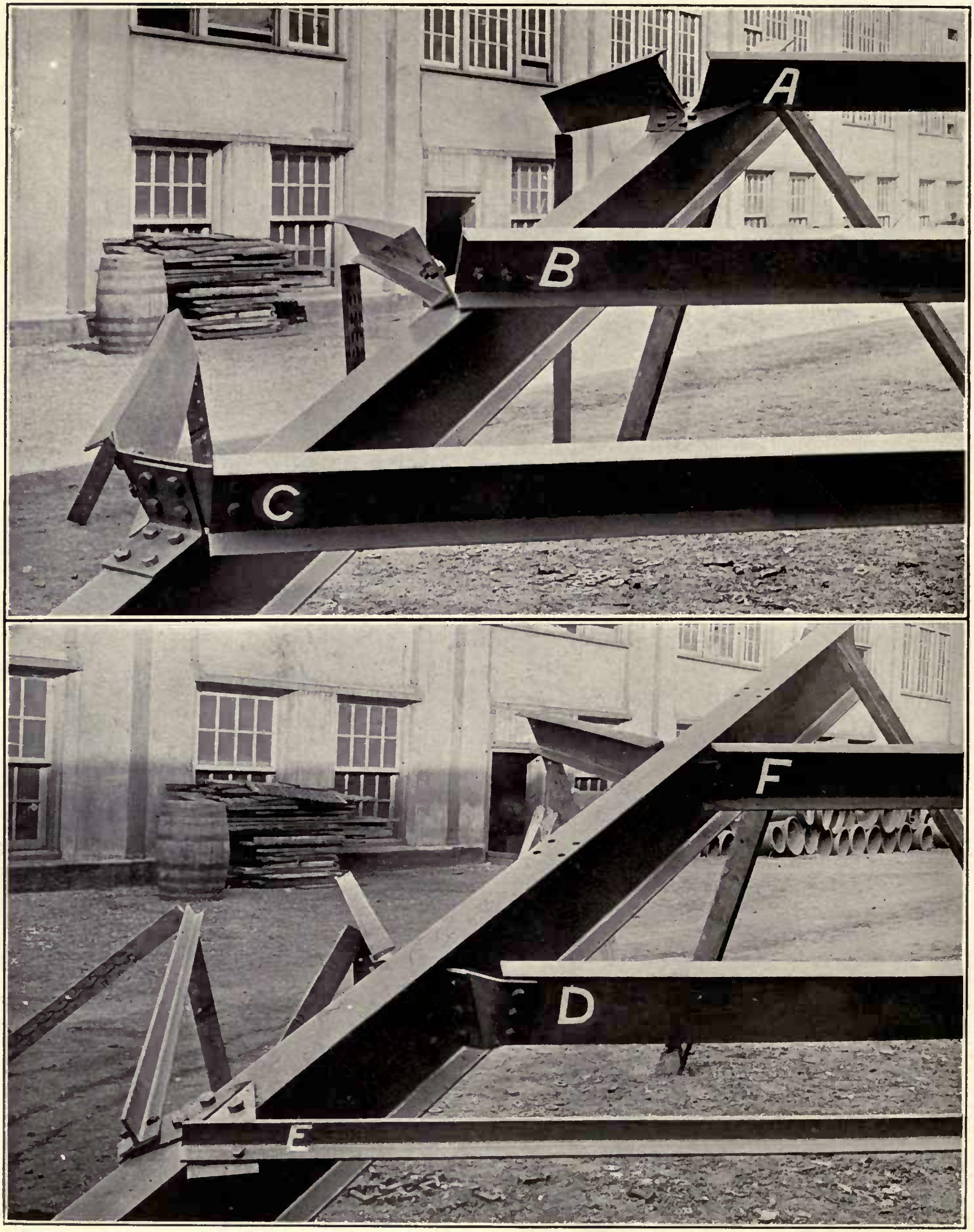
HIP AND VALLEY DESIGN

DETAILS, FORMULAE AND GRAPHICS

SIX STYLES OF HIP RAFTER CONNECTIONS



HIP AND VALLEY DESIGN

DETAILS, FORMULAE AND GRAPHICS

ROOFS

HOPPERS AND PIPE LINES

BY

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PREFACE.

The difficulty of making working shop drawings for roof connections at Hip and Valley is appreciated by Structural Engineers.

This book has been prepared to cover practical working details for such construction and to present the analytic and graphic processes needful for their development.

From the presentation of the designs here given, Engineers and Architects can determine the style of connection adapted to their demands readily and can specify the same for the structures they have in charge.

To Draftsmen the treatment of the subject will especially appeal, resulting to them in a saving of extra labor and concern.

Students will discover the practical training in descriptive geometry and trigonometry as applied to active engineering to be exceptionally valuable. Class room work in the proof of the formulae is recommended to Engineering Schools.

H. L. McKibben.

L. E. Gray.

Engineers with American Bridge Co.

FOREWORD.

On pages 3, 4 and 5 are shown working details for styles A, B, C, D, E and F, or six methods of connection to Hip Rafters from which to select the one that conforms best to the adjoining framing.

On pages 6, 7 and 8 are found working details for styles A, B, C, D, E and F, or six methods of similar connections to Valley Rafters from which to choose the one most desirable.

A sketch appears with each style of detail showing the position of the purlin in the main roof section, and small sub-formulae showing solutions for the variables y_1 to y_{10}, with special attention to y_1 and y_2.

After making selection of style desired, the detailer should solve the angles required as shown in details; i. e., $\angle 8$ and $\angle 9$ are needed in style C. No other angles need be found; only those involved in the style chosen.

Solution of these angles can be readily made from general formulae on page 10, if the worker be familiar with Trigonometry and Logarithms; if not, results may be obtained from the simple graphics given on pages 11, 12 and 13, making the problem easy for the detailer who is not familiar with formulae.

If the case in hand be one that is covered by the tabulated solutions on pages 14, 15 and 16, the worker can take from those tables any or all variables which develop in a roof of pitch $^1/_5$, $^1/_4$, $^1/_3$, 30° or 55°, if the angle B in plan is 30°, 45° or 50°.

These tabulated solutions give the values of the variables for designs in most common use without the necessity of solving any angles whatever; but the formulae on page 10 and graphics on pages 11, 12 and 13 furnish data for solving angles for any roof pitch and all possible positions of rafter.

In styles A, B, C and E the roof line being above the main truss metal line, the worker will need to use formulae on page 9 to locate working point "d."

The authors desire to call especial attention to the following:

1st. The known data are in all cases the main roof pitch or Angle A. The position of Hip or Valley Rafter, Angle B, which is the angle formed by rafter and main truss as seen in *Plan* looking directly perpendicular to lower side of Angle A. No other data than A and B as above described is ever required.

Throughout both details and graphics the letter "d" refers always to the same working point; the marks d_1 and d_2 refer also to this same point, viewed from different positions.

2d. All formulae on page 10 are logarithmic, and in terms of tangent functions.

3d. Use of the graphics on pages 11, 12 and 13 expedite the work and give accurate results.

4th. A short method of graphics for solution of Angles $\angle 5$, $\angle 6$ and $\angle 8$ also appears on page 10, which may be used after solving $\angle 3$ and $\angle 4$, if desired.

5th. For those desiring to follow out the proofs given on pages 21 to 29, the four major intersecting planes involved are as follows (see page 10):

ROOF PLANE.

Seen in Elevation of Truss as line ab.
Seen in Plan as inclined surface a_1, b_1, r_1.

PURLIN WEB PLANE.

Seen in Elevation of Truss as line c, d.
Seen in Plan as inclined surface d_1, c_1, e_1.

RAFTER WEB PLANE.

Seen in Elevation of Rafter as surface r_2, c_2, b_2.
Seen in Plan as line r_1, b_1.

RAFTER FLANGE PLANE.

Seen in Elevation of Rafter as line r_2, b_2.
Seen in Plan as inclined surface r_1, b_1, e_1.

6th. Other formulae which may be used if desired are as follows:

Cos $\angle 3$=Cos R Cos $\angle 1$ Sec A.
Tan $\angle 5$=Cos A Tan B Cos $\angle 1$.
Tan $\angle 5$=Tan $\angle 2$ Cos $\angle 1$.
Tan $\angle 7$=Sin A Sin B Cos $\angle 4$.
Tan $\angle 7$=Cos $\angle 2$ Tan $\angle 10$.

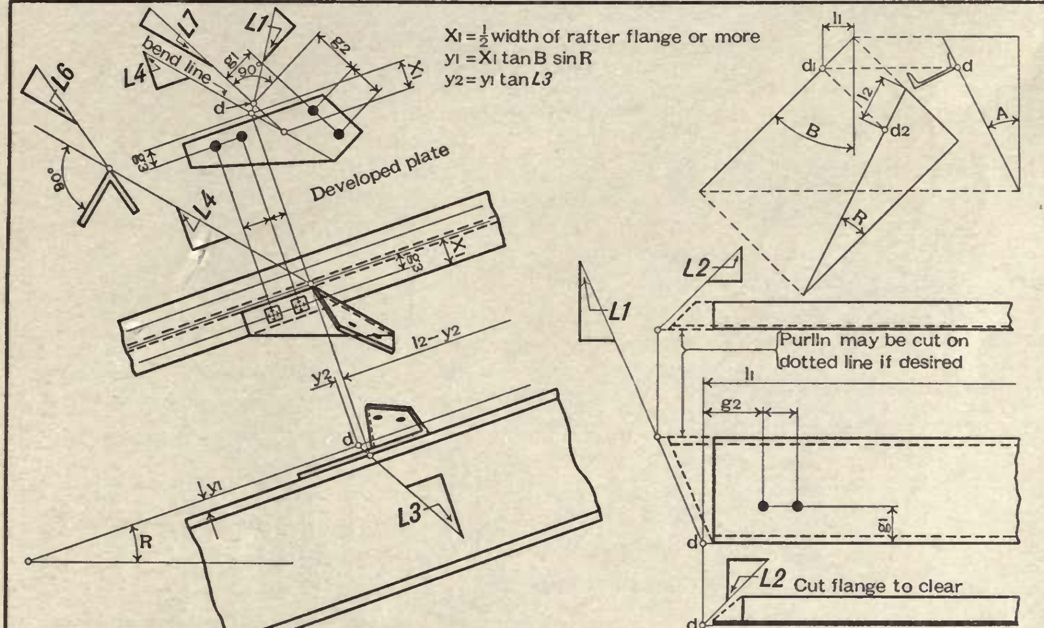
HOPPERS, BINS AND CHUTES (FORMS OF VALLEY CONSTRUCTION).

Details for these structures are left to the judgment of the detailer and are usually governed by the main design. The solution of the bend on connecting plate at dihedral intersections is the only difficulty for most draftsmen. Both formulae and graphics are provided on page 17 for ready use.

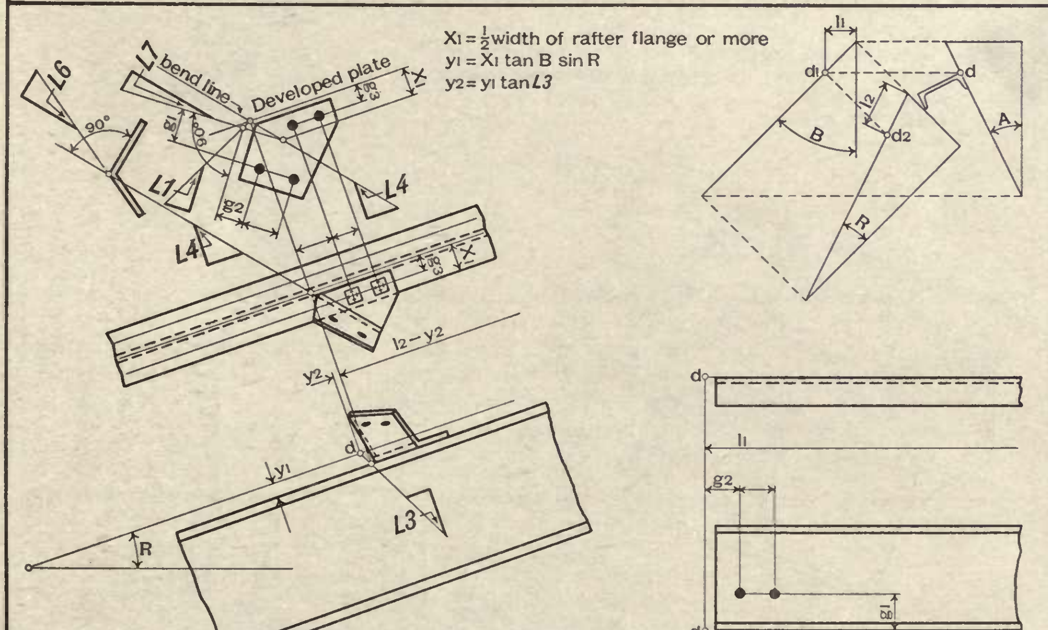
PIPE LINES.

Large Pipe Lines often require both horizontal and vertical change of direction at the same point, which condition may give rise to annoying details. Two separate bends are more expensive and produce greater friction on the flow than a single resultant bend. Careful attention to resultant angles *"X"* and detail angles *"Y"* will save much trouble in fabrication and improve the efficiency of the finished structure.

HIP RAFTER DETAILS

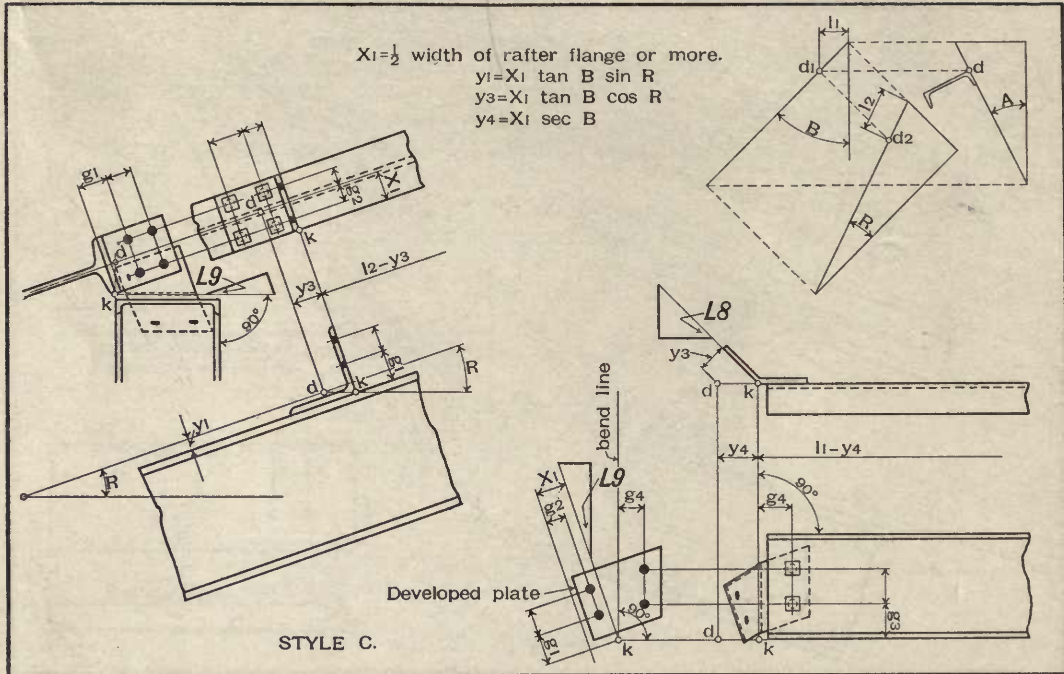


STYLE A.

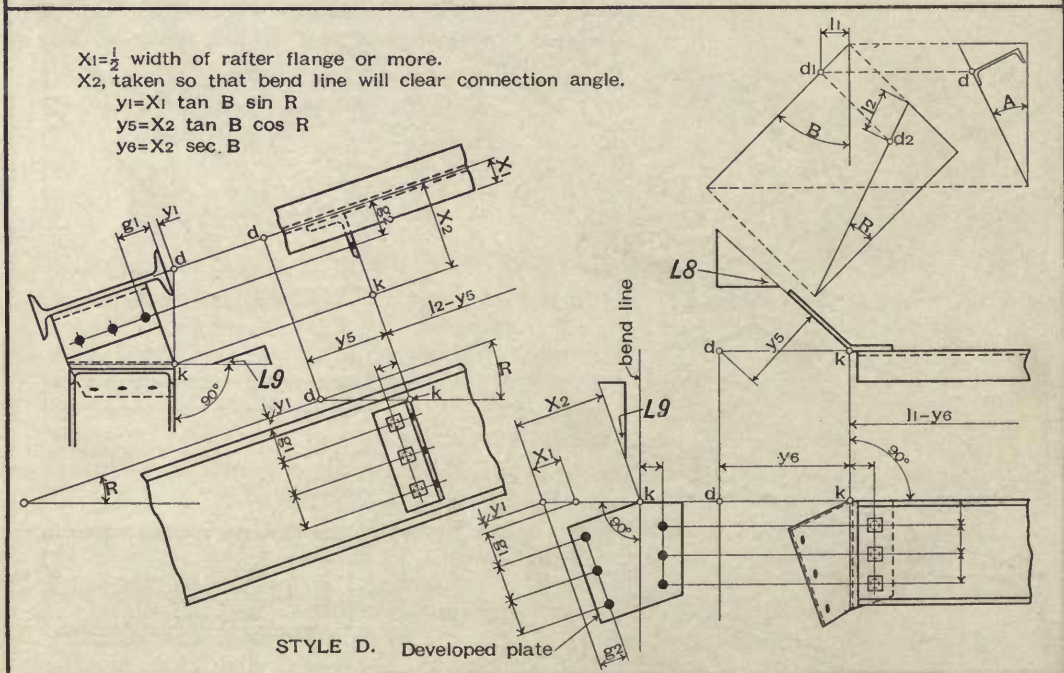


STYLE B.

HIP RAFTER DETAILS



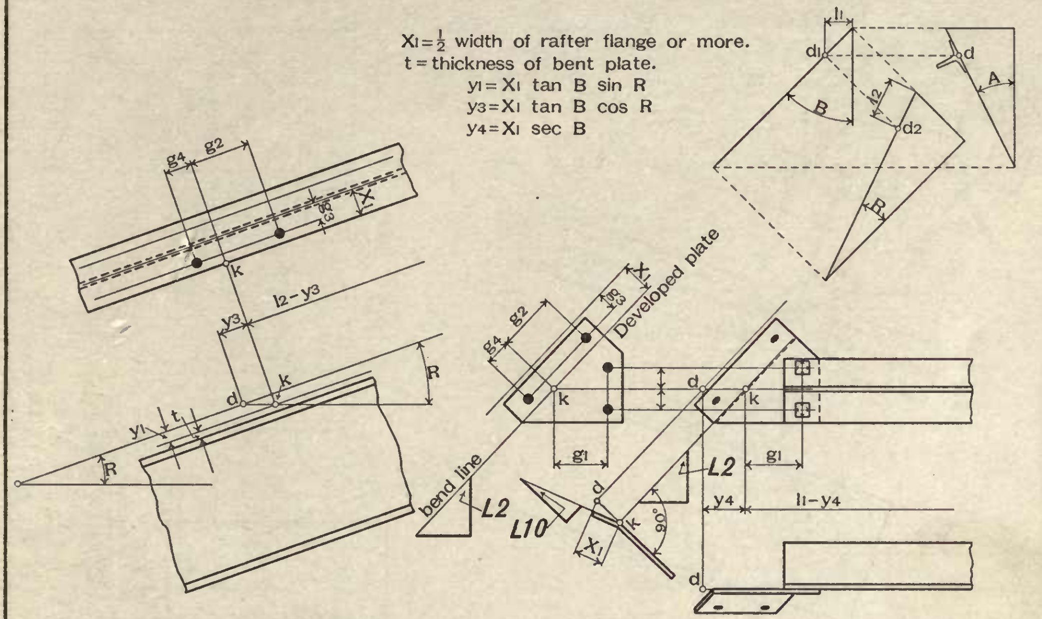
STYLE C.



STYLE D.

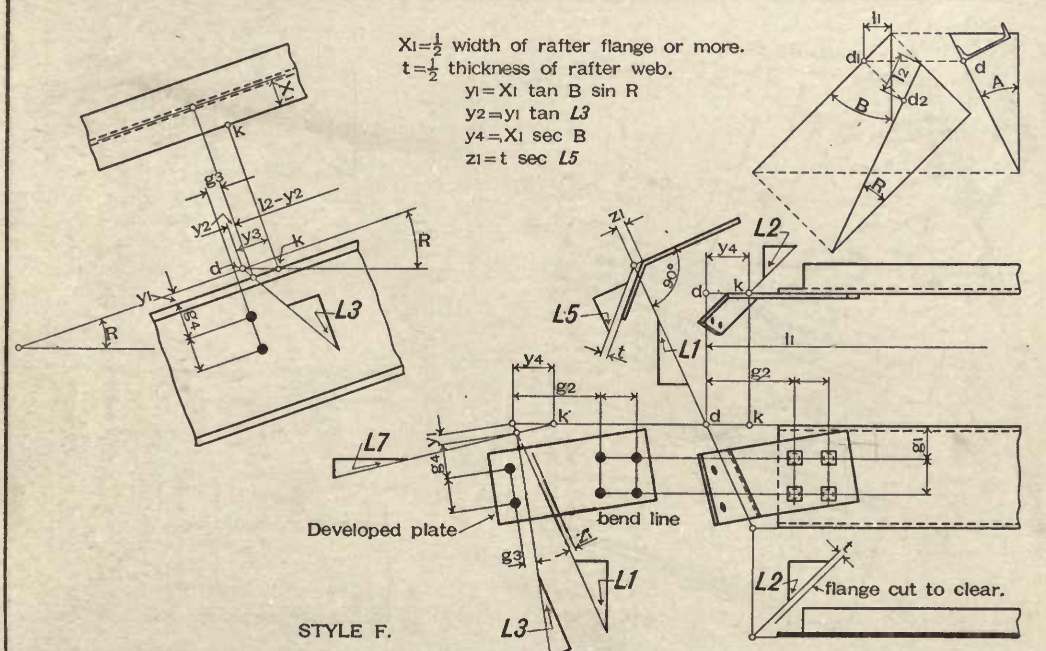
HIP RAFTER DETAILS

$X_1 = \frac{1}{2}$ width of rafter flange or more.
t = thickness of bent plate.
$y_1 = X_1 \tan B \sin R$
$y_3 = X_1 \tan B \cos R$
$y_4 = X_1 \sec B$



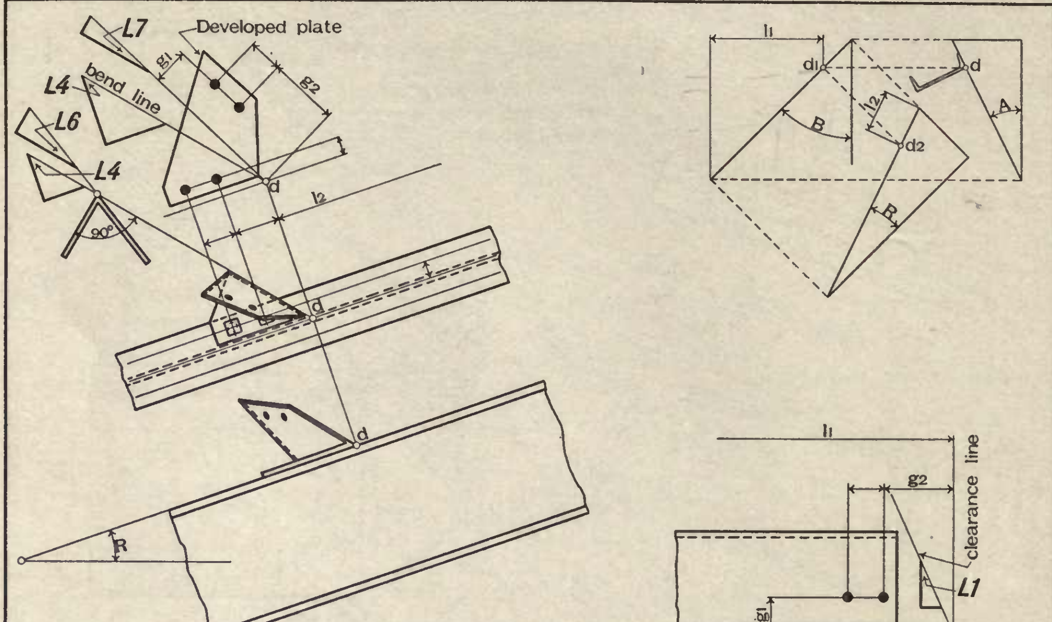
STYLE E.

$X_1 = \frac{1}{2}$ width of rafter flange or more.
$t = \frac{1}{2}$ thickness of rafter web.
$y_1 = X_1 \tan B \sin R$
$y_2 = y_1 \tan L3$
$y_4 = X_1 \sec B$
$z_1 = t \sec L5$

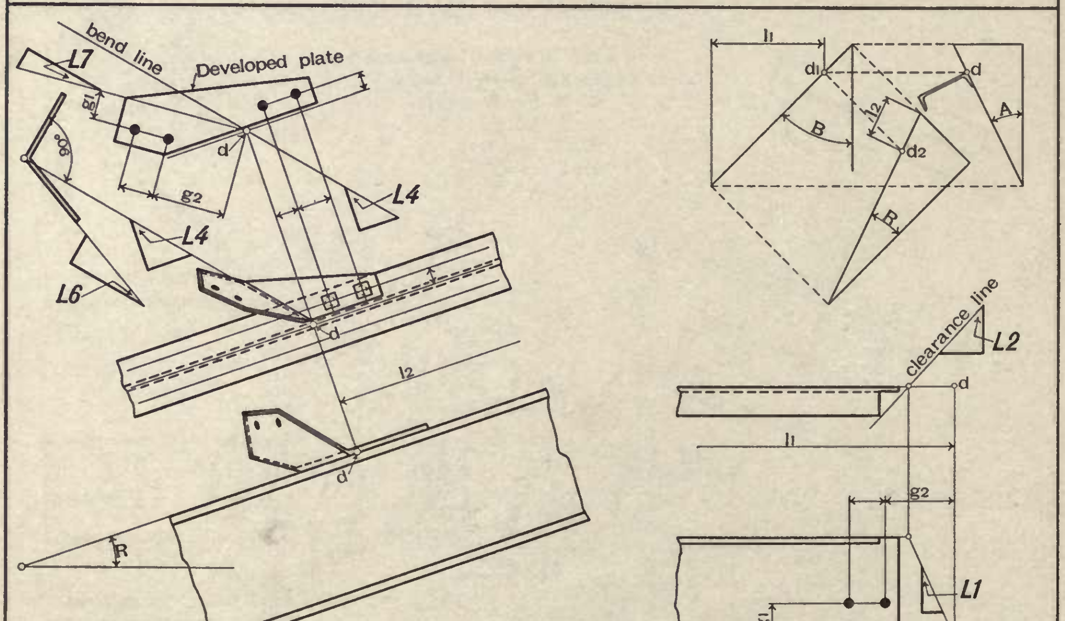


STYLE F.

VALLEY RAFTER DETAILS

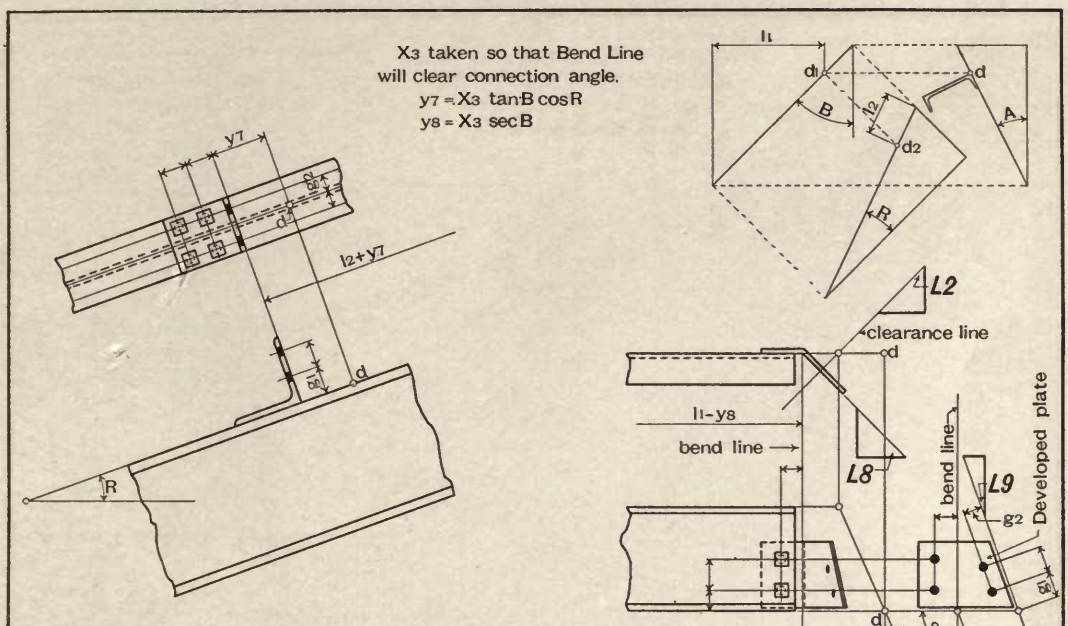


STYLE A.

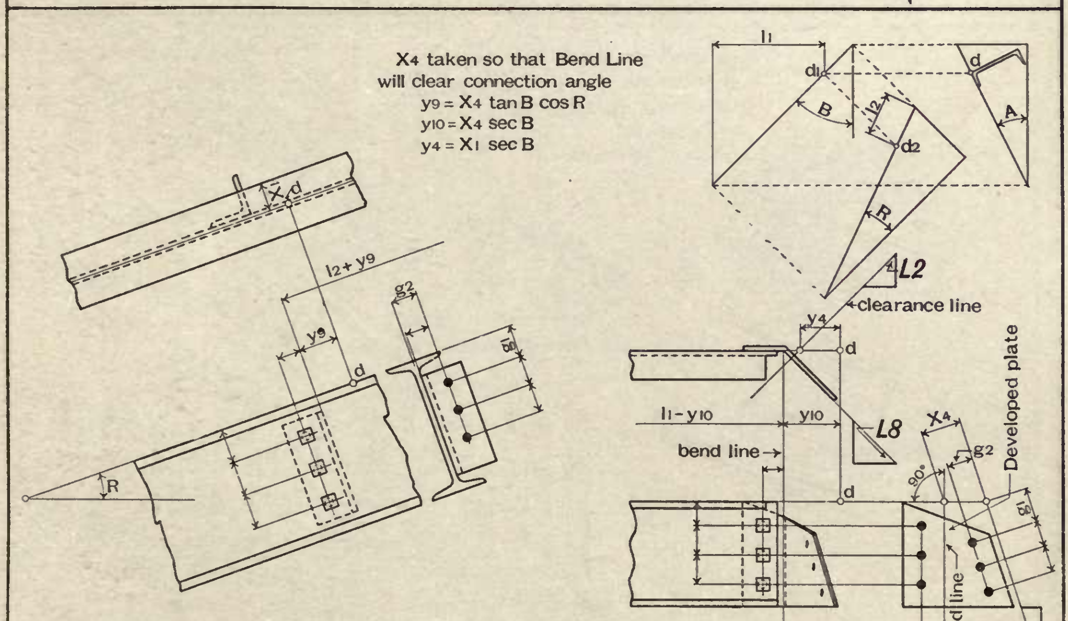


STYLE B.

VALLEY RAFTER DETAILS



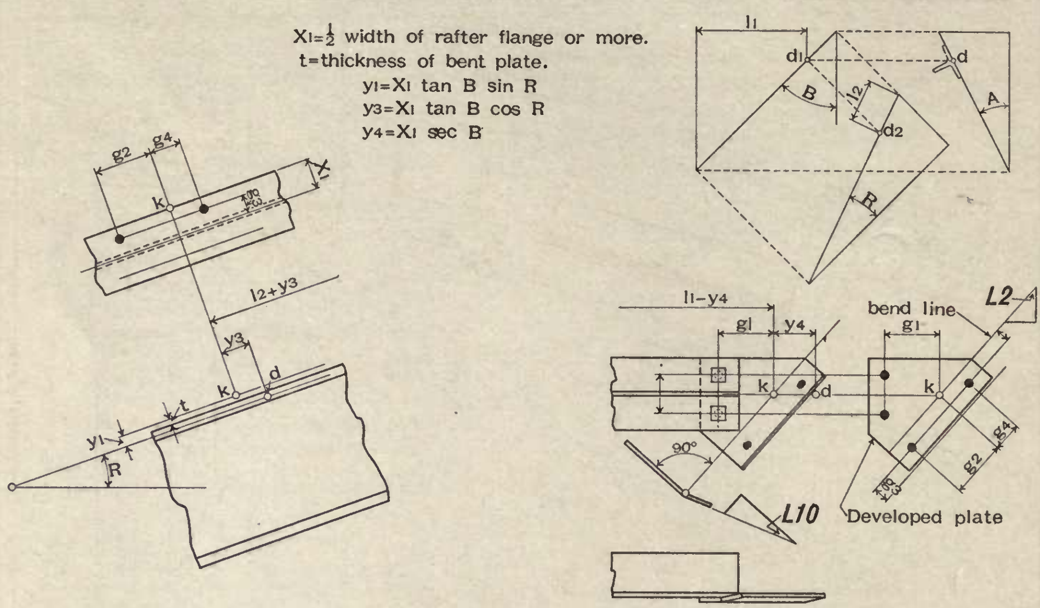
STYLE C.



STYLE D.

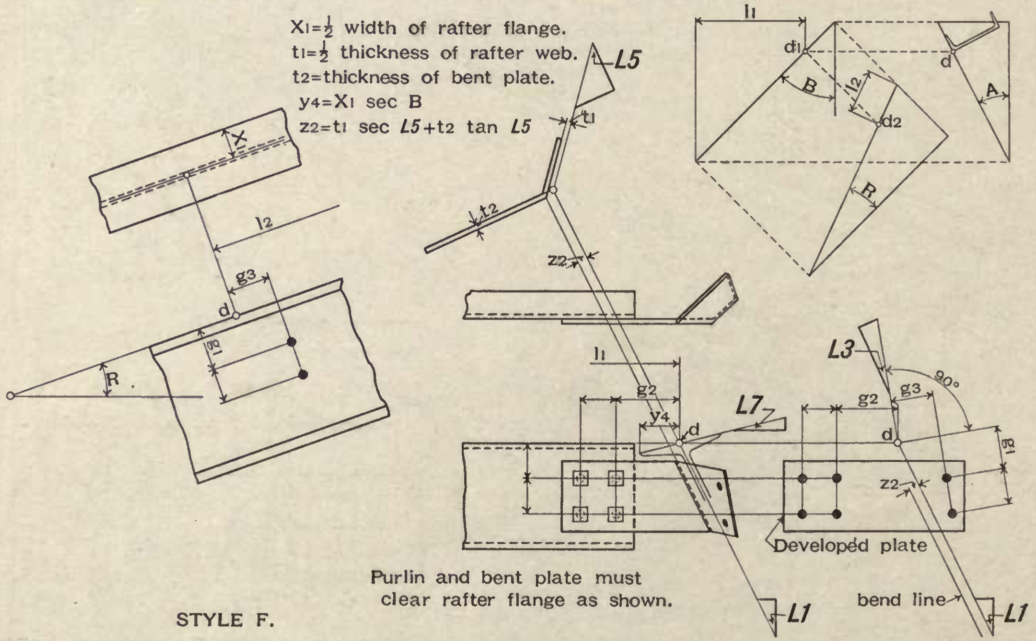
VALLEY RAFTER DETAILS

$X_1 = \frac{1}{2}$ width of rafter flange or more.
t=thickness of bent plate.
$y_1 = X_1 \tan B \sin R$
$y_3 = X_1 \tan B \cos R$
$y_4 = X_1 \sec B$



STYLE E.

$X_1 = \frac{1}{2}$ width of rafter flange.
$t_1 = \frac{1}{2}$ thickness of rafter web.
t_2=thickness of bent plate.
$y_4 = X_1 \sec B$
$z_2 = t_1 \sec \angle 5 + t_2 \tan \angle 5$

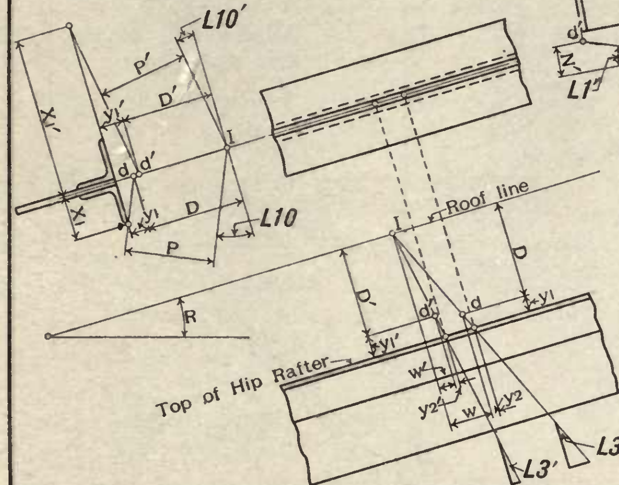


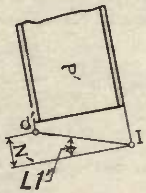
Purlin and bent plate must clear rafter flange as shown.

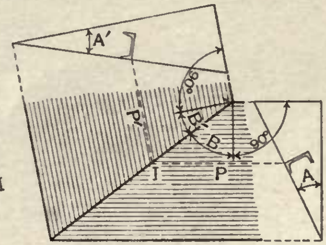
STYLE F.

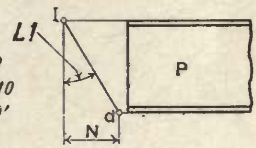
RELATIONS OF ROOF LINE TO WORKING POINTS USED IN FORMULAE

Given $\begin{cases} X_1 \text{ or } X_1' = \frac{1}{2} \text{ width of rafter flange or more.} \\ P = \text{actual depth of purlin.} \\ P' = \text{ " " " " } \end{cases}$









$D = P. \sec L10$

$y_1 = X_1 \tan L10$

$D' = P' \sec L10'$

$y_1' = D + y_1 - D'$

$X_1' = y_1' \cot L10'$

$y_2 = y_1 \tan L3$

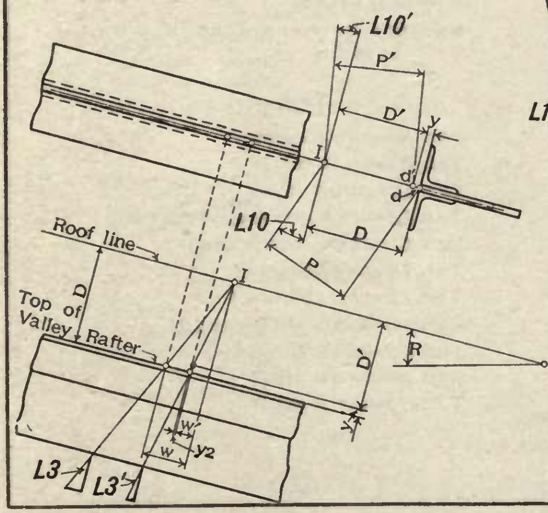
$y_2' = y_1' \tan L3'$

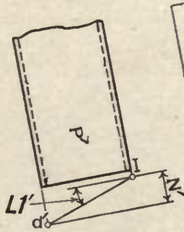
$w = D \tan L3$

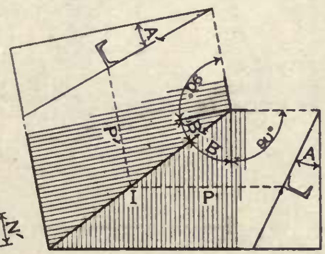
$w' = D' \tan L3'$

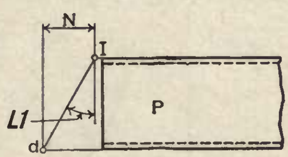
$N = P \tan L1$

$N' = P' \tan L1'$









$y = D - D'$

$y_2 = y \tan L3'$

$w = D \tan L3$

$w' = D' \tan L3'$

$N = P \tan L1$

$N' = P' \tan L1'$

GENERAL FORMULAE

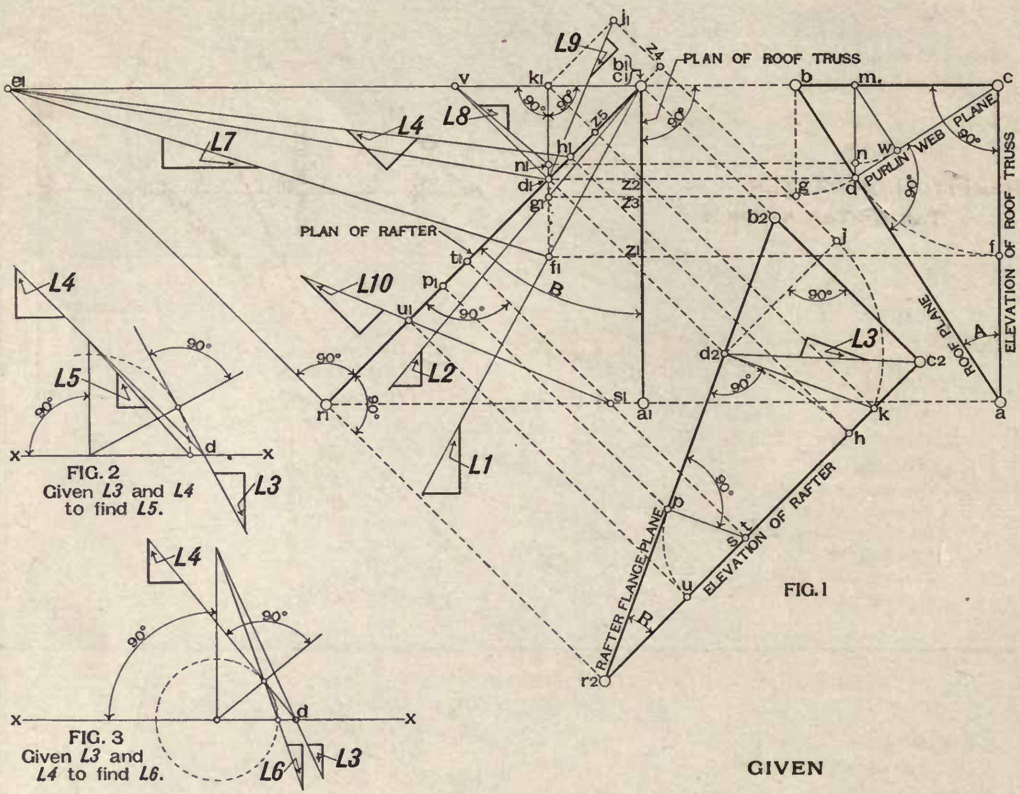


FIG. 1

FIG. 2
Given $L3$ and $L4$ to find $L5$.

FIG. 3
Given $L3$ and $L4$ to find $L6$.

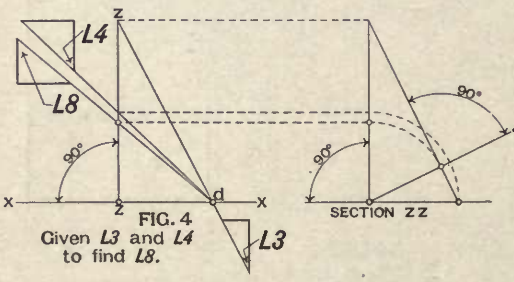


FIG. 4
Given $L3$ and $L4$ to find $L8$.

In figs. 2, 3 and 4, the line xx is the intersection of Rafter Flange and Rafter Web.

GIVEN

A=Pitch of Roof.

B=Angle between Truss and Rafter in plan.

FORMULAE

Tan R = tan A cos B

Tan $L1$ = sin A tan B

Tan $L2$ = cos A tan B

Tan $L3$ = sin A cos A sin B tan B

Tan $L4$ = cos² A tan B sec R

Tan $L5$ = cos $L3$ tan $L4$

Tan $L6$ = tan $L3$ cos $L4$

Tan $L7$ = tan B sin R cos $L2$

Tan $L8$ = cos A tan B

Tan $L9$ = tan B sin R

Tan $L10$ = tan B sin R

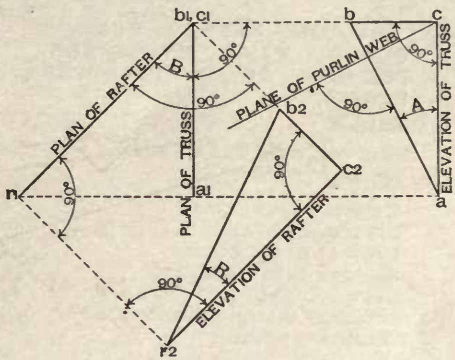
GRAPHIC SOLUTION OF ANGLES

A = PITCH OF ROOF

B = ANGLE BETWEEN TRUSS AND RAFTER IN PLAN

R = PITCH OF RAFTER

Tan R = Tan A Cos B



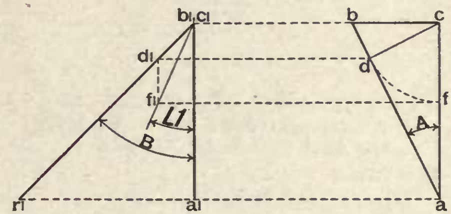
L1—BEVEL ON PURLIN WEB PLANE MADE BY INTERSECTION OF RAFTER WEB PLANE

FORMULA

Tan *L1* = Sin A Tan B

GRAPHICS

- Draw d, c ⊥ a, b
- Draw d, d1 ‖ b, b1
- Revolve d to f, about c
- Draw f, f1 ‖ d, d1
- Draw d1, f1 ⊥ d, d1
- Connect f1 with c1



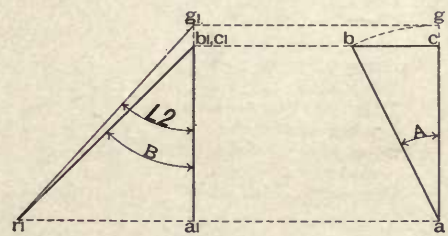
L2—BEVEL ON ROOF PLANE MADE BY INTERSECTION OF RAFTER WEB PLANE

FORMULA

Tan *L2* = Tan B Cos A

GRAPHICS

- Revolve b to g about a
- Draw g, g1 ‖ b, b1
- Extend a1, b1 to g1
- Connect g1 with r1



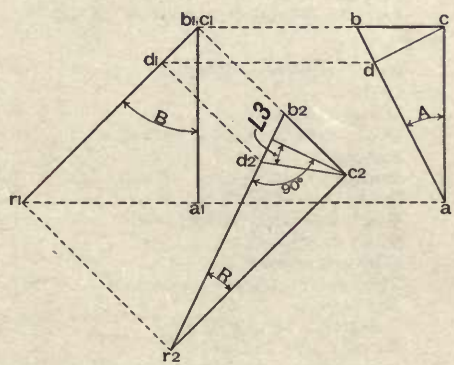
L3—BEVEL ON RAFTER WEB PLANE MADE BY INTERSECTION OF PURLIN WEB PLANE

FORMULA

Tan *L3* = Sin A Cos A Sin B Tan B

GRAPHICS

- Draw d, c ⊥ a, b
- Draw d, d1 ‖ b, b1
- Draw d1, d2 ‖ b1, b2
- Connect d2 with c2



GRAPHIC SOLUTION OF ANGLES

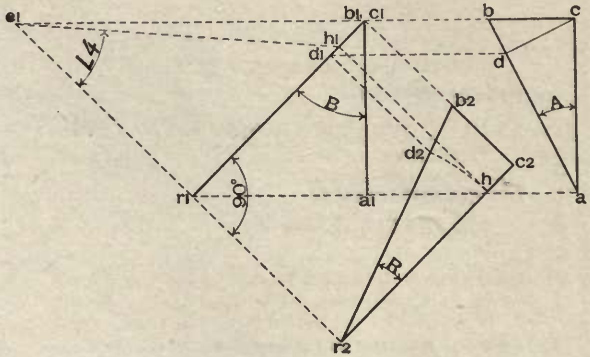
L4—BEVEL ON RAFTER FLANGE PLANE MADE BY INTERSECTION OF PURLIN WEB PLANE

FORMULA

Tan *L4* = Cos^2 A Tan B Sec R

GRAPHICS

Draw $d, c \perp a, b$
Draw $d, d_1 \parallel b, b_1$
Draw $d_1, d_2 \parallel b_1, b_2$
Revolve d_2 to h, about r_2
Draw $h, h_1 \parallel b_1, b_2$
Extend b, b_1 to intersect r_1, r_2 at e_1
Connect e_1 with h_1



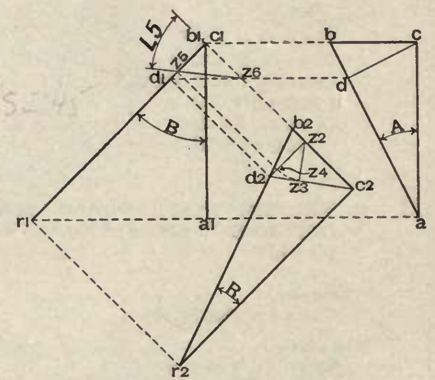
L5—COMPLEMENT OF ANGLE BETWEEN PURLIN WEB PLANE AND RAFTER WEB PLANE

FORMULA

Tan *L5* = Cos *L3* Tan *L4*

GRAPHICS

Draw $d, c \perp a, b$
Draw $d, d_1 \parallel b, b_1$
Draw $d_1, d_2 \parallel b_1, b_2$
Draw $d_2, z_2 \perp b_2, c_2$
Draw $z_2, z_3 \perp d_2, c_2$
Revolve z_3 to z_4 about z_2
Draw $z_4, z_5 \parallel b_1, b_2$
Locate z_6 at intersection of d, d_1 and c_1, c_2
Connect z_5 with z_6



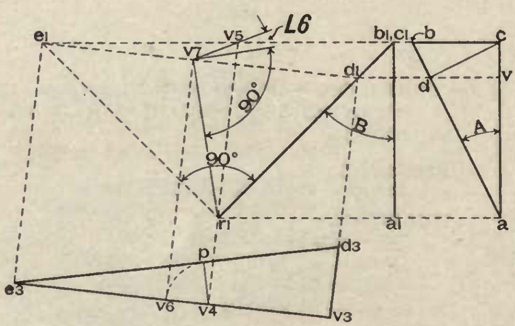
L6—COMPLEMENT OF ANGLE BETWEEN PURLIN WEB PLANE AND RAFTER FLANGE PLANE

FORMULA

Tan *L6* = Tan *L3* Cos *L4*

GRAPHICS

Draw $d, c \perp a, b$
Draw $d, d_1 \parallel b, b_1$ and extend to v
Extend b, b_1 to e_1
Connect e_1 with d_1
Draw $e_3, v_3 \parallel e_1, d_1$
Draw e_1, e_3 and $d_1, v_3 \perp e_1, d_1$
Take $v_3, d_3 = d, v$
Connect e_3 with d_3
Through r_1, draw $v_4, v_5 \perp e_1, d_1$
Draw $v_4, p \perp e_3, d_3$
Revolve p to v_6 about v_4
Draw $v_6, v_7 \perp e_1, d_1$
Connect v_7 with r_1 and v_5



GRAPHIC SOLUTION OF ANGLES

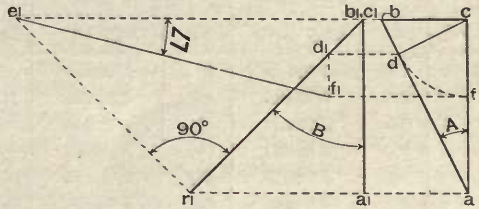
∠7—BEVEL ON PURLIN WEB PLANE MADE BY RAFTER FLANGE PLANE

FORMULA

Tan ∠7 Tan B Sin R Cos ∠2

GRAPHICS

Draw d, c ⊥ a, b
Draw d, d_1 ∥ b, b_1
Revolve d to f about c
Draw f, f_1 ∥ b, b_1
Draw d_1, f_1 ⊥ d, d_1
Extend b, b_1 to e_1
Connect e_1 with f_1



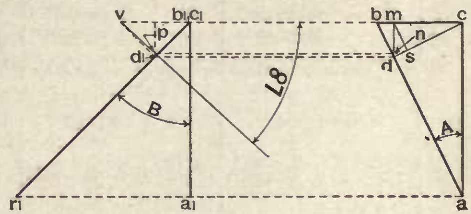
∠8—ANGLE BETWEEN PURLIN WEB PLANE AND A PLANE PERPENDICULAR TO BOTH RAFTER WEB PLANE AND RAFTER FLANGE PLANE

FORMULA

Tan ∠8 = Tan B, Cos A

GRAPHICS

Draw d, c ⊥ a, b
Draw d, d_1 ∥ b, b_1
Draw d, m ⊥ b, c
Draw m, s ⊥ d, c
Revolve s to n about m
Draw n, p ∥ d, d_1
Draw d_1 p ⊥ d, d_1
Draw d_1, v ⊥ r_1, b_1 to intersect b, b_1 at v
Connect v with p



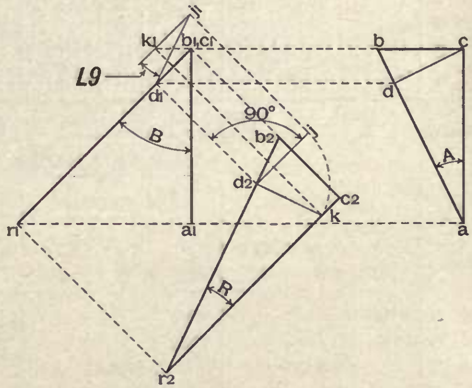
∠9—BEVEL ON PLANE PERPENDICULAR TO BOTH RAFTER WEB PLANE AND RAFTER FLANGE PLANE MADE BY INTERSECTION OF PURLIN WEB PLANE

FORMULA

Tan ∠9 = Tan B Sin R

GRAPHICS

Draw d, c ⊥ a, b
Draw d, d_1 ∥ b, b_1
Draw d_1, d_2 ∥ b_1, b_2
Draw d_2, k ⊥ r_2, b_2
Draw k, k_1 ⊥ r_1, b_1
Revolve k to j about d_2
Draw j, j_1 ∥ k, k_1
Draw k_1, j_1 ⊥ k, k_1
Connect d_1 with j_1



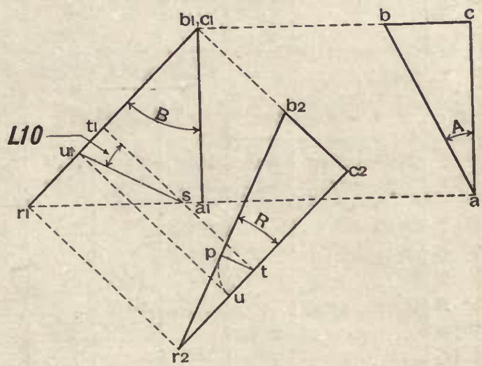
∠10—ANGLE BETWEEN ROOF PLANE AND RAFTER FLANGE PLANE

FORMULA

Tan ∠10 = Tan B Sin R

GRAPHICS

Take p any point on b_2, r_2
Draw p, t ⊥ b_2. r_2
Revolve p to u about t
Draw t, t_1 ∥ b_1, b_2
Draw u, u_1 ∥ b_1, b_2
Locate s at intersection of t, t_1 and a, r_1
Connect u_1 with s



SOLUTIONS, FIVE ORDINARY ROOF PITCHES

B=30°

A	1/5 PITCH		1/4 PITCH		30° PITCH		1/3 PITCH		55° PITCH	
	"S"	Log. Tan.	"S"	Log. Tan.	"S"	Log. Tan.	"S"	Log. Tan.	"S"	Log. Tan.
R	4 5/32	9.53959	5 3/16	9.63650	6	9.69897	6 15/16	9.76144	14 27/32	0.09230
L 1	2 9/16	9.33127	3 3/32	9.41195	3 15/32	9.46041	3 27/32	9.50550	5 11/16	9.67480
L 2	6 7/16	9.72921	6 3/16	9.71298	6	9.69897	5 3/4	9.68159	3 31/32	9.52003
L 3	1 3/16	8.99801	1 13/32	9.06247	1 1/2	9.09691	1 19/32	9.12462	1 5/8	9.13237
L 4	6 5/16	9.72159	6 1/32	9.70185	5 13/16	9.68496	5 17/32	9.66421	3 5/8	9.48016
L 5	6 9/32	9.71945	6	9.69897	5 3/4	9.68159	5 1/2	9.66039	3 19/32	9.47620
L 6	1 1/16	8.94484	1 1/4	9.01344	1 11/32	9.05119	1 7/16	9.08268	1 9/16	9.11340
L 7	2	9.22157	2 7/16	9.30929	2 25/32	9.36350	3 1/8	9.41532	5 1/8	9.62961
L 8	6 7/16	9.72921	6 3/16	9.71298	6	9.69897	5 3/4	9.68159	3 31/32	9.52003
L 9	2 9/32	9.27642	2 3/4	9.36062	3 3/32	9.41195	3 15/32	9.46041	5 5/8	9.65221
L 10	2 9/32	9.27642	2 3/4	9.36062	3 3/32	9.41195	3 15/32	9.46041	5 5/8	9.65221

"S" = Corresponding Bevels or Slopes to Base of 12 inches.

A	1/5 PITCH					1/4 PITCH					30° PITCH					1/3 PITCH					55° PITCH				
X 1	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4
Y 1	9/32	15/32	9/16	13/16	1 3/16	11/32	9/16	11/16	31/32	1 7/16	3/8	21/32	25/32	1 3/32	1 5/8	7/16	23/32	7/8	1 7/32	1 13/16	11/16	1 1/8	1 11/32	1 29/32	2 13/16
Y 2	1/32	1/16	1/16	3/32	1/8	1/32	1/16	3/32	1/8	5/32	1/16	3/32	3/32	1/8	3/16	1/16	3/32	1/8	5/32	1/4	3/32	5/32	3/16	1/4	3/8
Y 3	13/16	1 3/8	1 5/8	2 5/16	3 13/32	25/32	1 5/16	1 19/32	2 1/4	3 5/16	25/32	1 9/32	1 9/16	2 3/16	3 7/32	3/4	1 1/4	1 1/2	2 1/8	3 1/8	17/32	29/32	1 3/32	1 17/32	2 9/32
Y 4	1 23/32	2 7/8	3 15/32	4 29/32	7 7/32	1 23/32	2 7/8	3 15/32	4 29/32	7 7/32	1 23/32	2 7/8	3 15/32	4 29/32	7 7/32	1 23/32	2 7/8	3 15/32	4 29/32	7 7/32	1 23/32	2 7/8	3 15/32	4 29/32	7 7/32

For Purlins not exceeding 12″ Depth, with 4½″ Connection Clearance, the following assigned values of X_2, X_3, X_4, give good Results. Y_5 to Y_{10} derived therefrom.

CLEARANCES FOR 9 INCH PURLIN

A	1/5 PITCH			1/4 PITCH			30° PITCH			1/3 PITCH			55° PITCH		
	X_2= 6½	X_3= 5	X_4= 4½	X_2= 6¾	X_3= 5	X_4= 4½	X_2= 7	X_3= 5	X_4= 4½	X_2= 7¼	X_3= 5	X_4= 4½	X_2= 8½	X_3= 5	X_4= 4½
Y 5	3 17/32	....	....	3 9/16	....	....	3 5/8	....	....	3 5/8	....	....	3 3/32	....	....
Y 6	7 1/2	....	....	7 25/32	....	....	8 3/32	....	....	8 3/8	....	....	9 13/16	....	....
Y 7	....	2 23/32	....	....	2 21/32	....	....	2 19/32	....	....	2 1/2	....	....	1 13/16	....
Y 8	....	5 25/32	....	....	5 25/32	....	....	5 25/32	....	....	5 25/32	....	....	5 25/32	....
Y 9	....	....	2 15/32	....	....	2 3/8	....	....	2 5/16	....	....	2 1/4	....	....	1 5/8
Y10	....	....	5 3/16	....	....	5 3/16	....	....	5 3/16	....	....	5 3/16	....	....	5 3/16

CLEARANCES FOR 12 INCH PURLIN

A	1/5 PITCH			1/4 PITCH			30° PITCH			1/3 PITCH			55° PITCH		
	X_2= 7	X_3= 7½	X_4= 4½	X_2= 7½	X_3= 7½	X_4= 4½	X_2= 8	X_3= 7½	X_4= 4½	X_2= 8½	X_3= 7½	X_4= 4½	X_2= 10	X_3= 7½	X_4= 4½
Y 5	3 13/16	....	....	3 31/32	....	....	4 1/8	....	....	4 1/4	....	....	3 5/8	....	....
Y 6	8 3/32	....	....	8 21/32	....	....	9 1/4	....	....	9 13/16	....	....	11 9/16	....	....
Y 7	....	4 3/32	....	....	3 31/32	....	....	3 7/8	....	....	3 3/4	....	....	2 23/32	....
Y 8	....	8 21/32	....	....	8 21/32	....	....	8 21/32	....	....	8 21/32	....	....	8 21/32	....
Y 9	....	....	2 15/32	....	....	2 3/8	....	....	2 5/16	....	....	2 1/4	....	....	1 5/8
Y10	....	....	5 3/16	....	....	5 3/16	....	....	5 3/16	....	....	5 3/16	....	....	5 3/16

SOLUTIONS, FIVE ORDINARY ROOF PITCHES

B=45°

A	1/5 PITCH		1/4 PITCH		30° PITCH		1/3 PITCH		55° PITCH	
	"S"	Log. Tan.	"S"	Log. Tan.	"S"	Log. Tan.	"S"	Log. Tan.	"S"	Log. Tan.
R	3 13/32	9.45154	4 1/4	9.54845	4 7/8	9.61092	5 21/32	9.67339	12 1/8	0.00426
L 1	4 15/32	9.56983	5 3/8	9.65051	6	9.69897	6 21/32	9.74406	9 27/32	9.91336
L 2	11 5/32	9.96777	10 3/4	9.95154	10 13/32	9.93753	10	9.92015	6 7/8	9.75859
L 3	2 15/16	9.38709	3 3/8	9.45154	3 11/16	9.48599	3 29/32	9.51369	4	9.52144
L 4	10 3/4	9.95225	10 3/16	9.92867	9 23/32	9.90853	9 3/16	9.88387	5 5/8	9.66984
L 5	10 7/16	9.93971	9 13/16	9.91195	9 5/16	9.88908	8 23/32	9.86190	5 5/16	9.64710
L 6	2 3/16	9.25914	2 19/32	9.33379	2 27/32	9.37641	3 1/8	9.41357	3 5/8	9.47851
L 7	2 13/32	9.29984	2 31/32	9.39524	3 7/16	9.45593	3 15/16	9.51558	7 13/32	9.78984
L 8	11 5/32	9.96777	10 3/4	9.95154	10 13/32	9.93753	10	9.92015	6 7/8	9.75859
L 9	3 1/4	9.43483	4	9.52288	4 17/32	9.57745	5 1/8	9.62982	8 17/32	9.85160
L 10	3 1/4	9.43483	4	9.52288	4 17/32	9.57745	5 1/8	9.62982	8 17/32	9.85160

"S" = Corresponding Bevels or Slopes to Base of 12 inches.

A	1/5 PITCH					1/4 PITCH					30° PITCH					1/3 PITCH					55° PITCH				
X 1	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4
Y 1	13/32	11/16	13/16	1 5/32	1 11/16	1/2	27/32	1	1 13/32	2 3/32	9/16	15/16	1 1/8	1 19/32	2 3/8	5/8	1 1/16	1 9/32	1 13/16	2 21/32	1 1/16	1 25/32	2 1/8	3 1/32	4 7/16
Y 2	3/32	5/32	3/16	9/32	13/32	5/32	1/4	9/32	13/32	19/32	3/16	9/32	11/32	1/2	23/32	7/32	11/32	13/32	19/32	7/8	11/32	19/32	23/32	1	1 15/32
Y 3	1 7/16	2 13/32	2 7/8	4 3/32	6	1 13/32	2 11/32	2 27/32	4	5 29/32	1 3/8	2 5/16	2 25/32	3 15/16	5 25/32	1 11/32	2 1/4	2 23/32	3 27/32	5 21/32	1 1/16	1 3/4	2 1/8	3	4 13/32
Y 4	2 1/8	3 17/32	4 1/4	6	8 27/32	2 1/8	3 17/32	4 1/4	6	8 27/32	2 1/8	3 17/32	4 1/4	6	8 27/32	2 1/8	3 17/32	4 1/4	6	8 27/32	2 1/8	3 17/32	4 1/4	6	8 27/32

For Purlins not exceeding 12″ Depth, with 4½″ Connection Clearance, the following assigned values of X_2, X_3, X_4 give good Results. Y_5 to Y_{10} derived therefrom.

CLEARANCES FOR 9 INCH PURLIN

A	1/5 PITCH			1/4 PITCH			30° PITCH			1/3 PITCH			55° PITCH		
	X_2= 7	X_3= 5½	X_4= 4½	X_2= 7½	X_3= 5½	X_4= 4½	X_2= 8	X_3= 5½	X_4= 4½	X_2= 8½	X_3= 5½	X_4= 4½	X_2= 10	X_3= 5½	X_4= 4½
Y 5	6 3/4	....	....	7 1/16	....	....	7 13/32	....	....	7 11/16	....	....	7 1/32	....	....
Y 6	9 29/32	....	....	10 19/32	....	....	11 5/16	....	....	12 1/32	....	....	14 5/32	....	....
Y 7	....	5 9/32	....	....	5 3/16	....	....	5 3/32	....	....	4 31/32	....	....	3 7/8	....
Y 8	....	7 25/32	....	....	7 25/32	....	....	7 25/32	....	....	7 25/32	....	....	7 25/32	....
Y 9	....	....	4 11/32	....	....	4 1/4	....	....	4 5/32	....	....	4 1/16	....	....	3 5/32
Y10	....	....	6 3/8	....	....	6 3/8	....	....	6 3/8	....	....	6 3/8	....	....	6 3/8

CLEARANCES FOR 12 INCH PURLIN

A	1/5 PITCH			1/4 PITCH			30° PITCH			1/3 PITCH			55° PITCH		
	X_2= 8	X_3= 7½	X_4= 4½	X_2= 8½	X_3= 7½	X_4= 4½	X_2= 9	X_3= 7½	X_4= 4½	X_2= 9½	X_3= 7½	X_4= 4½	X_2= 11½	X_3= 7½	X_4= 4½
Y 5	7 11/16	....	....	8	....	....	8 11/32	....	....	8 19/32	....	....	8 3/32	....	....
Y 6	11 5/16	....	....	12 1/32	....	....	12 3/4	....	....	13 7/16	....	....	16 1/4	....	....
Y 7	....	7 7/32	....	....	7 1/16	....	....	6 15/16	....	....	6 25/32	....	....	5 9/32	....
Y 8	....	10 19/32	....	....	10 19/32	....	....	10 19/32	....	....	10 19/32	....	....	10 19/32	....
Y 9	....	....	4 11/32	....	....	4 1/4	....	....	4 5/32	....	....	4 1/16	....	....	3 5/32
Y10	....	....	6 3/8	....	....	6 3/8	....	....	6 3/8	....	....	6 3/8	....	....	6 3/8

SOLUTIONS, FIVE ORDINARY ROOF PITCHES

B=50°

A	1/5 PITCH		1/4 PITCH		30° PITCH		1/3 PITCH		55° PITCH	
	"S"	Log. Tan.	"S"	Log. Tan.	"S"	Log. Tan.	"S"	Log. Tan.	"S"	Log. Tan.
R	3 3/32	9.41013	3 27/32	9.50704	4 15/32	9.56951	5 5/32	9.63198	11	9.96284
L 1	5 5/16	9.64602	6 13/32	9.72670	7 5/32	9.77516	7 15/16	9.82024	11 23/32	9.98955
L 2	13 9/32	0.04396	12 25/32	0.02773	12 3/8	0.01372	11 29/32	9.99634	8 3/16	9.83478
L 3	3 25/32	9.49804	4 3/8	9.56250	4 3/4	9.59694	5 1/16	9.62465	5 5/32	9.63240
L 4	12 23/32	0.02563	12 1/32	0.00062	11 7/16	9.97927	10 25/32	9.95309	6 3/8	9.72610
L 5	12 5/32	0.00511	11 9/32	9.97344	10 5/8	9.94775	9 15/16	9.91761	5 7/8	9.68942
L 6	2 19/32	9.33433	3 3/32	9.41167	3 7/16	9.45655	3 3/4	9.49632	4 17/32	9.57824
L 7	2 3/8	9.29881	3	9.39706	3 15/32	9.46025	4	9.52286	8	9.82305
L 8	13 9/32	0.04396	12 25/32	0.02773	12 3/8	0.01372	11 29/32	9.99634	8 3/16	9.83478
L 9	3 9/16	9.47241	4 3/8	9.56188	4 31/32	9.61767	5 5/8	9.67155	9 21/32	9.90630
L 10	3 9/16	9.47241	4 3/8	9.56188	4 31/32	9.61767	5 5/8	9.67155	9 21/32	9.90630

"S"=Corresponding Bevels or Slopes to Base of 12 inches.

A	1/5 PITCH					1/4 PITCH					30° PITCH					1/3 PITCH					55° PITCH				
X 1	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4	1 1/2	2 1/2	3	4 1/4	6 1/4
Y 1	7/16	3/4	29/32	1 1/4	1 27/32	9/16	29/32	1 3/32	1 9/16	2 9/32	5/8	1 1/32	1 1/4	1 3/4	2 19/32	23/32	1 3/16	1 13/32	2	2 15/16	1 7/32	2	2 13/32	3 7/16	5 1/32
Y 2	5/32	1/4	9/32	13/32	19/32	3/16	11/32	13/32	9/16	27/32	1/4	13/32	1/2	11/16	1 1/32	5/16	1/2	19/32	27/32	1 1/4	17/32	7/8	1 1/32	1 15/32	2 5/32
Y 3	1 23/32	2 7/8	3 15/32	4 29/32	7 7/32	1 11/16	2 27/32	3 13/32	4 13/16	7 3/32	1 11/16	2 25/32	3 11/32	4 3/4	6 31/32	1 21/32	2 3/4	3 9/32	4 21/32	6 27/32	1 5/16	2 3/16	2 5/8	3 23/32	5 1/2
Y 4	2 11/32	3 7/8	4 21/32	6 5/8	9 23/32	2 11/32	3 7/8	4 21/32	6 5/8	9 23/32	2 11/32	3 7/8	4 21/32	6 5/8	9 23/32	2 11/32	3 7/8	4 21/32	6 5/8	9 23/32	2 11/32	3 7/8	4 21/32	6 5/8	9 23/32

For Purlins not exceeding 12″ Depth, with 4½″ Connection Clearance, the following assigned values of X_2, X_3, X_4, give good results. Y_5 to Y_{10} derived therefrom.

CLEARANCES FOR 9 INCH PURLIN

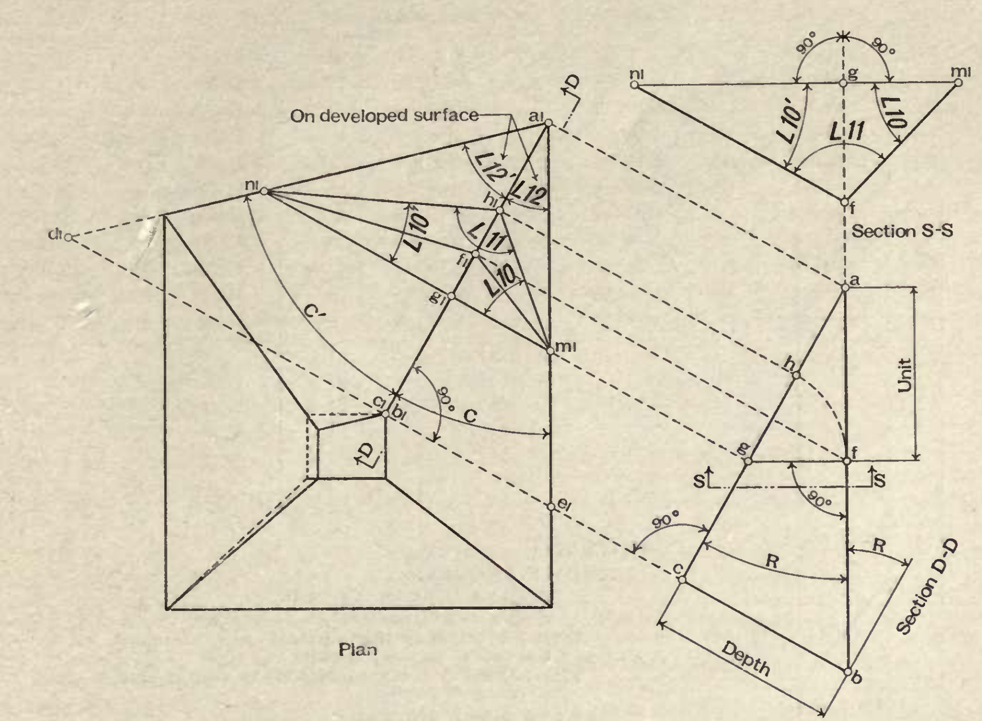
A	1/5 PITCH			1/4 PITCH			30° PITCH			1/3 PITCH			55° PITCH		
	X_2= 7½	X_3= 5½	X_4= 4½	X_2= 8	X_3= 5½	X_4= 4½	X_2= 8½	X_3= 5½	X_4= 4½	X_2= 9	X_3= 5½	X_4= 4½	X_2= 10½	X_3= 5½	X_4= 4½
Y 5	8 21/32	....	....	9 1/16	....	....	9 1/2	....	....	9 27/32	....	....	9 7/32	....	....
Y 6	11 21/32	....	....	12 7/16	....	....	13 7/32	....	....	14	....	....	16 11/32	....	....
Y 7	....	6 11/32	....	....	6 1/4	....	....	6 5/32	....	....	6 1/32	....	....	4 27/32	....
Y 8	....	8 9/16	....	....	8 9/16	....	....	8 9/16	....	....	8 9/16	....	....	8 9/16	....
Y 9	....	....	5 3/16	....	....	5 3/32	....	....	5 1/32	....	....	4 15/16	....	....	3 15/16
Y10	....	....	7	....	....	7	....	....	7	....	....	7	....	....	7

CLEARANCES FOR 12 INCH PURLIN

A	1/5 PITCH			1/4 PITCH			30° PITCH			1/3 PITCH			55° PITCH		
	X_2= 8½	X_3= 7½	X_4= 4½	X_2= 9	X_3= 7½	X_4= 4½	X_2= 9½	X_3= 7½	X_4= 4½	X_2= 10	X_3= 7½	X_4= 4½	X_2= 12½	X_3= 7½	X_4= 4½
Y 5	9 13/16	....	....	10 7/32	....	....	10 5/8	....	....	10 31/32	....	....	10 31/32	....	....
Y 6	13 7/32	....	....	14	....	....	14 25/32	....	....	15 9/16	....	....	19 7/16	....	....
Y 7	....	8 21/32	....	....	8 1/2	....	....	8 3/8	....	....	8 7/32	....	....	6 19/32	....
Y 8	....	11 21/32	....	....	11 21/32	....	....	11 21/32	....	....	11 21/32	....	....	11 21/32	....
Y 9	....	....	5 3/16	....	....	5 3/32	....	....	5 1/32	....	....	4 15/16	....	....	3 15/16
Y10	....	....	7	....	....	7	....	....	7	....	....	7	....	....	7

HOPPERS, BINS AND CHUTES

FORMULAE AND GRAPHICS FOR SOLUTION OF ANGLES



EXPLANATION

Inclined surfaces a_1, b_1, e_1, and a_1, b_1, d_1, intersect on line a_1, b_1, forming dihedral angle measured by angle $L11$. (See Section S-S.)

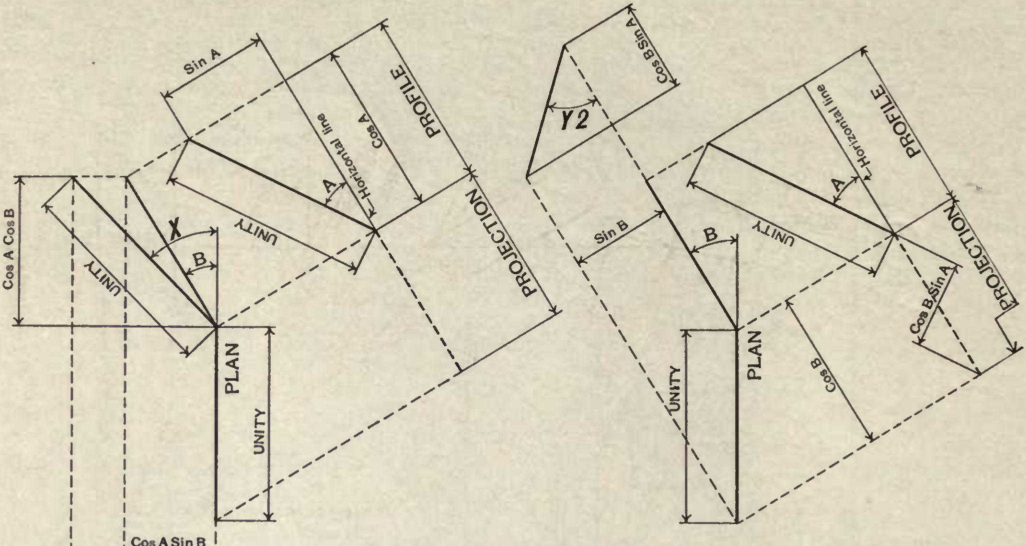
Vertical section a, b, c, (Section D-D) divides the dihedral into two dihedrals, of which $L10$ and $L10'$ are respectively the complements.

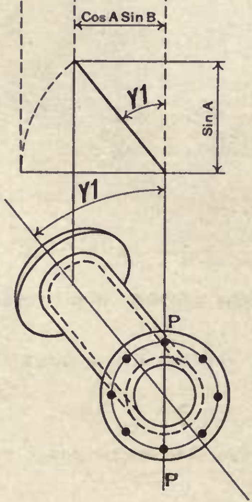
Angles R, C and C′ must be determined from design.

Rectangular bottom with irregular top will produce slightly warped side surfaces, see dotted lines for this condition.

GRAPHICS	FORMULAE
Choose any point f in line a, b	$\tan L10 = \sin R \cot C$
Draw f, g, $\perp$ a, b	$\tan L10' = \sin R \cot C'$
Draw g, n_1 $\perp$ a, c	$L11 = 180° - (L10 + L10')$
Project f to f_1 in plan	$\tan L12 = \sec R \tan C \sec L10$
Draw f_1, m_1 and f_1, n_1 in plan	$\tan L12' = \sec R \tan C' \sec L10'$
Then m_1, n_1, f_1 is plan of Section S-S	$\cos L12 = \cos R \cos C$
Revolve f to h about g	$\cos L12' = \cos R \cos C'$
Project h to h_1 in plan	
Draw h_1, m_1, and h_1, n_1	
Then h_1, m_1, n_1 is true view of Section S–S	

PIPE CONNECTION, RESULTANT OF TWO BENDS





NOTES:—

KNOWN ANGLES:—

Angle A in plane of profile

Angle B in horizontal plane or plan

THE PROFILE is the vertical section taken thru the center of the pipe line

THE LINE P-P is perpendicular to center line of pipe in plane of profile

ANGLES TO BE SOLVED:—

Resultant Angle X

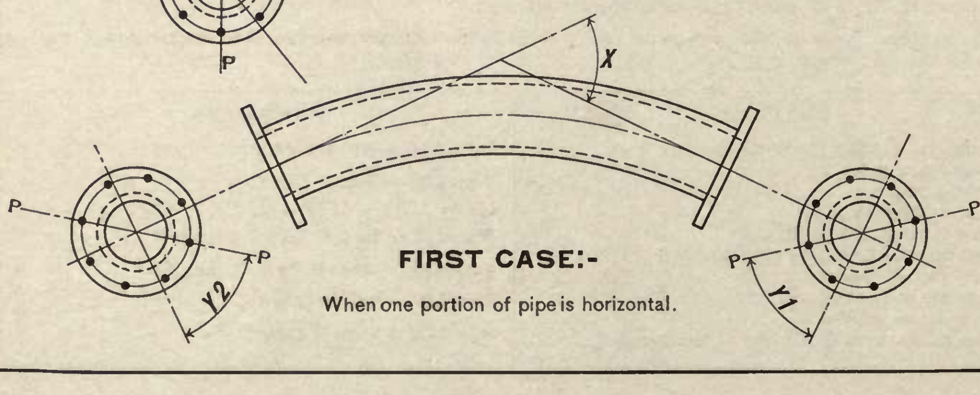
Detail Angles $Y1$ and $Y2$

FORMULAE:—

$$\text{Cos } X = \text{Cos A Cos B}$$

$$\text{Tan } Y1 = \frac{\text{Cos A Sin B}}{\text{Sin A}} = \text{Cot A Sin B}$$

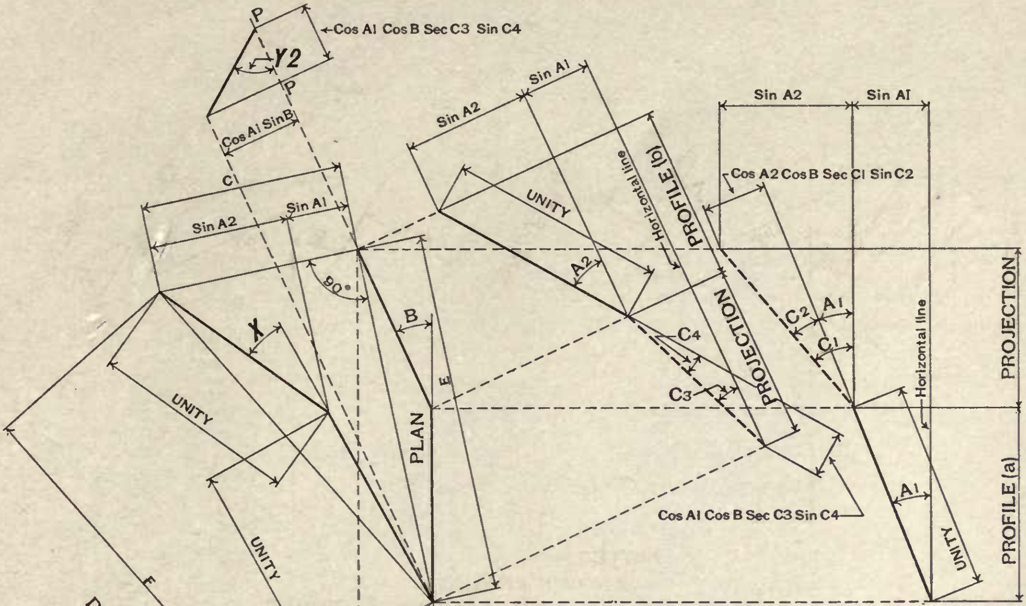
$$\text{Tan } Y2 = \frac{\text{Sin B}}{\text{Cos B Sin A}} = \text{Tan B Cosec A}$$

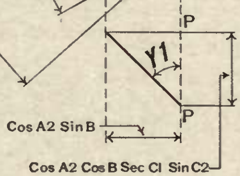


FIRST CASE:-

When one portion of pipe is horizontal.

PIPE CONNECTION, RESULTANT OF TWO BENDS





KNOWN ANGLES:—

Angle A_1 in plane of profile (a)
Angle A_2 in plane of profile (b)
Angle B in horizontal plane or plan

ANGLES USED:—

$$\operatorname{Tan} C_1 = \frac{\operatorname{Sin} A_2}{\operatorname{Cos} A_2 \operatorname{Cos} B} = \operatorname{Tan} A_2 \operatorname{Sec} B$$

$$C_2 = C_1 - A_1$$

$$\operatorname{Tan} C_3 = \frac{\operatorname{Sin} A_1}{\operatorname{Cos} A_1 \operatorname{Cos} B} = \operatorname{Tan} A_1 \operatorname{Sec} B$$

$$C_4 = A_2 - C_3$$

ANGLES TO BE SOLVED:—

Resultant Angle X
Detail Angles $Y1$ and $Y2$

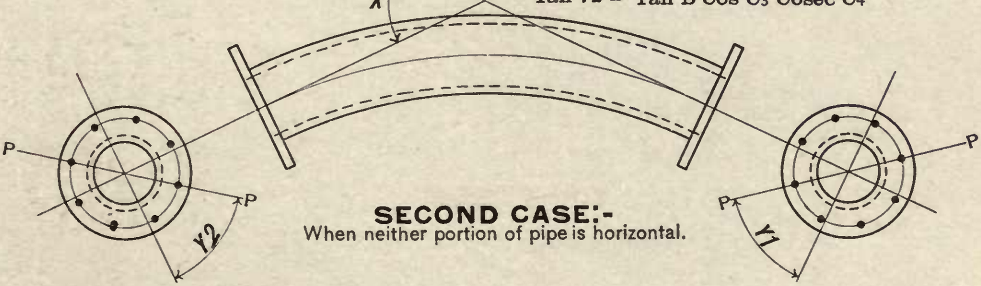
FORMULAE:—

$$\operatorname{Cos} X = \operatorname{Cos} A_1 \operatorname{Cos} A_2 \operatorname{Cos} B + \operatorname{Sin} A_1 \operatorname{Sin} A_2$$

$$\text{or, } \operatorname{Cos} X = \frac{\operatorname{Sin} A_2 \operatorname{Cos} C_2}{\operatorname{Sin} C_1}$$

$$\operatorname{Tan} Y1 = \operatorname{Tan} B \operatorname{Cos} C_1 \operatorname{Cosec} C_2$$

$$\operatorname{Tan} Y2 = \operatorname{Tan} B \operatorname{Cos} C_3 \operatorname{Cosec} C_4$$



SECOND CASE:-
When neither portion of pipe is horizontal.

Refer to Page 10; a c = unity.

Tangent R

$\text{Tan } R = \dfrac{c_2, b_2}{c_2, r_2}$

$c_2, b_2 = c, b$

$= \text{Tan A}$

$c_2, r_2 = c', r'$

$= \text{Sec B}$

$\therefore \text{Tan R} = \dfrac{\text{Tan A}}{\text{Sec B}}$

$= \text{Tan A Cos B}$

Tangent $\angle 1$

$\text{Tan } \angle 1 = \dfrac{f', z'}{f, c}$

$f', z' = d', z_2$

$= (d, m) \text{ Tan B}$

But $d, m = \text{Sin}^2\text{A}$

$\therefore f', z' = \text{Sin}^2 \text{ A Tan B}$

$f, c = d, c$

$= \text{Sin A}$

$\therefore \text{Tan } \angle 1 = \dfrac{\text{Sin}^2 \text{ A Tan B}}{\text{Sin A}}$

$= \text{Sin A Tan B}$

Tangent $\angle 2$

$\text{Tan } \angle 2 = \dfrac{g', z_3}{b, g}$

$g', z_3 = d', z_2$

$= (d, m) \text{ Tan B}$

But $d, m = \text{Sin}^2 \text{ A}$

$\therefore g', z_3 = \text{Sin}^2 \text{ A Tan B}$

$b, g = b, d$

$= (d, c) \text{ Tan A}$

$= \text{Sin A Tan A}$

$\therefore \text{Tan } \angle 2 = \dfrac{\text{Sin}^2 \text{ A Tan B}}{\text{Sin A Tan A}}$

$= \dfrac{\text{Sin A Tan B}}{\text{Tan A}}$

$= \text{Cos A Tan B}$

Tangent $\angle 4$

$\text{Tan } \angle 4 = \dfrac{r', h'}{r', e'}$

$r', h' = r_2, h$

$= r_2, d_2$

$= \text{Cos}^2 \text{ A Sec B Sec R}$

$r', e' = \text{Csc B}$

$\therefore \text{Tan } \angle 4 = \dfrac{\text{Cos}^2 \text{ A Sec B Sec R}}{\text{Csc B}}$

$= \dfrac{\text{Cos}^2 \text{ A Sin B}}{\text{Cos B Cos R}}$

$= \text{Cos}^2 \text{ A Tan B Sec R}$

Tangent $\angle 7$

$\text{Tan } \angle 7 = \dfrac{f', k'}{e', k'}$

$f', k' = c, f$

$= d, c$

$= \text{Sin A}$

$a, d = \text{Cos A}$

$r_2, d_2 = (a, d) \text{ Sec } \angle 2$

$= \text{Cos A Sec } \angle 2$

$r_2, k = (r_2, d_2) \text{ Sec R}$

$= \text{Cos A Sec } \angle 2 \text{ Sec R}$

$e', k' = (r_2, k) \text{ Csc B}$

$= \text{Cos A Sec } \angle 2 \text{ Sec R Csc B}$

$\therefore \text{Tan } \angle 7 = \dfrac{\text{Sin A}}{\text{Cos A Sec } \angle 2 \text{ Sec R Csc B}}$

$= \text{Tan A Cos } \angle 2 \text{ Cos R Sin B}$

$= \text{Tan A} \left(\dfrac{\text{Sin R}}{\text{Tan R}}\right) \text{Tan B Cos B Cos} \angle 2$

$= \dfrac{\text{Tan A Sin R Tan B Cos B Cos} \angle 2}{\text{Tan A Cos B}}$

$= \text{Sin R Tan B Cos } \angle 2$

Tangent $\angle 8$

$\text{Tan } \angle 8 = \dfrac{n', k'}{k', v}$

$n', k' = n, m$

$= m, w$

$= (d, m) \text{ Cos A}$

$= \text{Sin}^2 \text{ A Cos A}$

$k', v = (d', k') \text{ Cot B}$

$= (d, m) \text{ Cot B}$

$= \text{Sin}^2 \text{ A Cot B}$

$\therefore \text{Tan } \angle 8 = \dfrac{\text{Sin}^2 \text{ A Cos A}}{\text{Sin}^2 \text{ A Cot B}}$

$= \text{Cos A Tan B}$

Tangent $\angle 9$

$\text{Tan } \angle 9 = \dfrac{j', z_4}{d', z_4}$

$d', z_4 = d_2, j$

$= d_2, k$

$j', z_4 = k', z_5$

$= (d', z_5) \text{ Tan B}$

$d', z_5 = (d_2, k) \text{ Sin R}$

$\therefore j', z_4 = (d_2, k) \text{ Sin R Tan B}$

$\therefore \text{Tan } \angle 9 = \dfrac{(d_2, k) \text{ Sin R Tan B}}{(d_2, k)}$

$= \text{Sin R Tan B}$

Tangent $\angle 10$

$\text{Tan } \angle 10 = \dfrac{u', t'}{t', s'}$

p is any point on r_2, b_2

Choose location such that s' will fall at a

$t', s' = \text{Sin B}$

$t, r_2 = t', r'$

$= (r', s') \text{ Sin B}$

$= \text{Tan B Sin B}$

$u', t' = u, t$

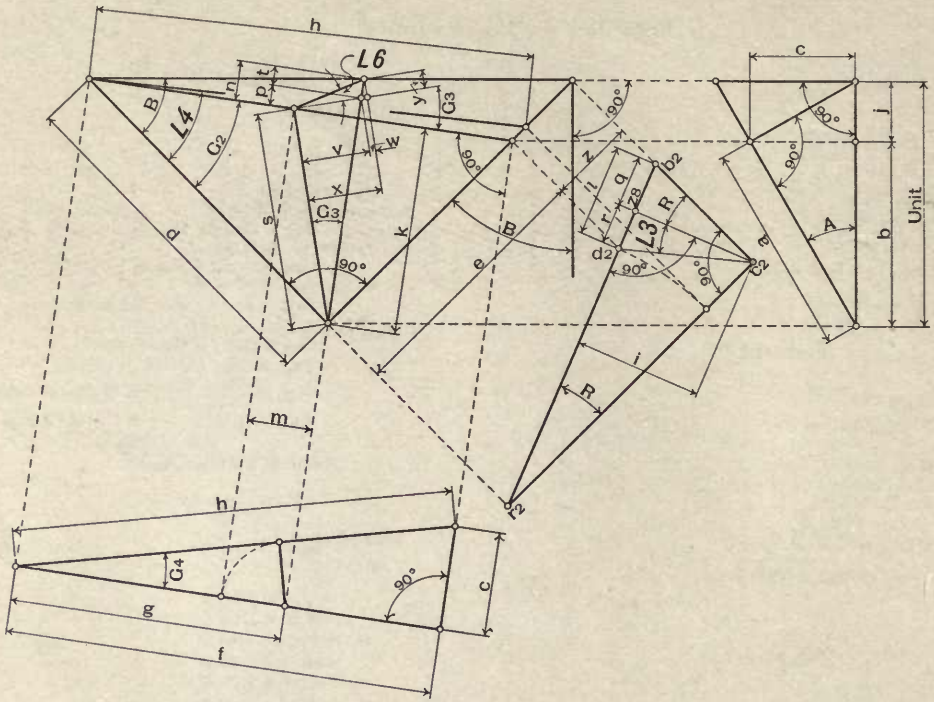
$= p, t$

$= (t, r_2) \text{ Sin R}$

$= \text{Tan B Sin B Sin R}$

$\therefore \text{Tan } \angle 10 = \dfrac{\text{Tan B Sin B Sin R}}{\text{Sin B}}$

$= \text{Tan B Sin R}$



Tangent $\angle 3$

1. $\text{Tan } \angle 3 = \dfrac{r}{l}$

2. $= \dfrac{l - q}{l}$

3. $j = \text{Sin}^2 A$

4. $b_2, c_2 = \text{Tan } A$

5. $z = \dfrac{j}{\text{Cos } B}$

6. $= \dfrac{\text{Sin}^2 A}{\text{Cos } B}$

7. $l = \dfrac{z}{\text{Cos } R}$

8. $= \dfrac{\text{Sin}^2 A}{\text{Cos } B \text{ Cos } R}$

9. $q = (b_2, c_2) \text{ Sin } R$

10. $= \text{Tan } A \text{ Sin } R$

11. $= \dfrac{\text{Sin } A \text{ Sin } R}{\text{Cos } A}$

12. $r = l - q$

13. $= \dfrac{\text{Sin}^2 A}{\text{Cos } B \text{ Cos } R} - \dfrac{\text{Sin } A \text{ Sin } R}{\text{Cos } A}$

14. $(e + z) = \dfrac{1}{\text{Cos } B}$

15. $l = (e + z) \text{ Sin } R$

16. $= \dfrac{1}{\text{Cos } B} \text{ Sin } R$

17. $= \dfrac{\text{Sin } R}{\text{Cos } B}$

18. $\therefore \text{Tan } \angle 3 = \dfrac{\dfrac{\text{Sin}^2 A}{\text{Cos } B \text{ Cos } R} - \dfrac{\text{Sin } A \text{ Sin } R}{\text{Cos } A}}{\dfrac{\text{Sin } R}{\text{Cos } B}}$

ANALYTIC PROOFS

19. $$=\frac{\dfrac{\mathrm{Sin^2A\ Cos\ A-Sin\ A\ Sin\ R\ Cos\ B\ Cos\ R}}{\mathrm{Cos\ B\ Cos\ R\ Cos\ A}}}{\dfrac{\mathrm{Sin\ R}}{\mathrm{Cos\ B}}}$$

20. $$=\frac{\mathrm{Sin^2A\ Cos\ A-Sin\ A\ Cos^2R\ Tan\ R\ Cos\ B}}{\mathrm{Cos\ A\ Cos^2R\ Tan\ R}}$$

21. $$=\frac{\mathrm{Sin^2A\ Cos\ A-Sin\ A\ Cos^2R\ Tan\ A\ Cos^2B}}{\mathrm{Cos\ A\ Cos^2R\ Tan\ A\ Cos\ B}}$$

22. $$=\frac{\mathrm{Cos\ A\ Sin^2A}-\dfrac{\mathrm{Sin^2A\ Cos^2R\ Cos^2B}}{\mathrm{Cos\ A}}}{\dfrac{\mathrm{Cos\ A\ Cos^2R\ Sin\ A\ Cos\ B}}{\mathrm{Cos\ A}}}$$

23. $$=\frac{\dfrac{\mathrm{Cos^2A\ Sin^2A-Sin^2A\ Cos^2R\ Cos^2B}}{\mathrm{Cos\ A}}}{\dfrac{\mathrm{Cos\ A\ Cos^2R\ Sin\ A\ Cos\ B}}{\mathrm{Cos\ A}}}$$

24. $$=\frac{\mathrm{Cos^2A\ Sin^2A-Sin^2A\ Cos^2R\ Cos^2B}}{\mathrm{Cos\ A\ Cos^2R\ Sin\ A\ Cos\ B}}$$

25. But, r_2, b_2 $$=\sqrt{\frac{1}{\mathrm{Cos^2B}}+\mathrm{Tan^2A}}$$

26. $$=\sqrt{\frac{1}{\mathrm{Cos^2B}}+\frac{\mathrm{Sin^2A}}{\mathrm{Cos^2A}}}$$

27. $$=\sqrt{\frac{\mathrm{Cos^2A+Sin^2A\ Cos^2B}}{\mathrm{Cos^2A\ Cos^2B}}}$$

28. $$=\frac{\sqrt{\mathrm{Sin^2A\ Cos^2B+Cos^2A}}}{\mathrm{Cos\ A\ Cos\ B}}$$

29. $$\mathrm{Cos\ R}=\frac{\dfrac{1}{\mathrm{Cos\ B}}}{\dfrac{\sqrt{\mathrm{Sin^2A\ Cos^2B+Cos^2A}}}{\mathrm{Cos\ A\ Cos\ B}}}$$

30. $$=\frac{\mathrm{Cos\ A}}{\sqrt{\mathrm{Sin^2A\ Cos^2B+Cos^2A}}}$$

Hence by substitution in No. 24.

31. $$\mathrm{Tan}\ \angle 3=\frac{\mathrm{Sin^2A\ Cos^2A-Sin^2A}\left(\dfrac{\mathrm{Cos^2A}}{\mathrm{Sin^2A\ Cos^2B+Cos^2A}}\right)\mathrm{Cos^2B}}{\mathrm{Cos\ A}\left(\dfrac{\mathrm{Cos^2A}}{\mathrm{Sin^2A\ Cos^2B+Cos^2A}}\right)\mathrm{Sin\ A\ Cos\ B}}$$

32. $$=\frac{\dfrac{\mathrm{Sin^2A\ Cos^2A\ (Sin^2A\ Cos^2B+Cos^2A)-Sin^2A\ Cos^2A\ Cos^2B}}{\mathrm{Sin^2A\ Cos^2B+Cos^2A}}}{\dfrac{\mathrm{Cos^3A\ Sin\ A\ Cos\ B}}{\mathrm{Sin^2A\ Cos^2B+Cos^2A}}}$$

33. $$=\frac{\mathrm{Sin\ A(Sin^2A\ Cos^2B+Cos^2A)-Sin\ A\ Cos^2B}}{\mathrm{Cos\ A\ Cos\ B}}$$

34. $$=\frac{\mathrm{Sin\ A(Sin^2A\ Cos^2B+Cos^2A-Cos^2B)}}{\mathrm{Cos\ A\ Cos\ B}}$$

35. $$=\frac{\mathrm{Sin\ A\ [Cos^2B\ (Sin^2A-1)+Cos^2A]}}{\mathrm{Cos\ A\ Cos\ B}}$$

36. $$=\frac{\mathrm{Sin\ A\ [Cos^2B(-Cos^2A)+Cos^2A]}}{\mathrm{Cos\ A\ Cos\ B}}$$

37. $$=\frac{\mathrm{Sin\ A(Cos^2A-Cos^2A\ Cos^2B)}}{\mathrm{Cos\ A\ Cos\ B}}$$

38. $$=\frac{\mathrm{Sin\ A\ Cos^2A(1-Cos^2B)}}{\mathrm{Cos\ A\ Cos\ B}}$$

39. $$=\frac{\mathrm{Sin\ A\ Cos^2A\ Sin^2B}}{\mathrm{Cos\ A\ Cos\ B}}$$

40. $$=\mathrm{Sin\ A\ Cos\ A\ Sin\ B\ Tan\ B}$$

ANALYTIC PROOFS

Tangent $\angle 6$—Refer to Page 22.

1. $a=\text{Cos A}$
2. $b=\text{Cos}^2\text{A}$
3. $c=\text{Cos A Sin A}$
4. $d=\dfrac{1}{\text{Sin B}}$
5. $e=\dfrac{\text{Cos}^2\text{A}}{\text{Cos B}}$
6. $f=\sqrt{d^2+e^2}$
7. $=\dfrac{\sqrt{\text{Sin}^2\text{B Cos}^4\text{A}+\text{Cos}^2\text{B}}}{\text{Cos B Sin B}}$
8. Let $M=\sqrt{\text{Cos}^2\text{B}+\text{Cos}^4\text{A Sin}^2\text{B}}$ (for convenience)
9. Then $f=\dfrac{M}{\text{Cos B Sin B}}$
10. $h=\sqrt{c^2+f^2}$
11. $=\sqrt{\text{Cos}^2\text{A Sin}^2\text{A}+\dfrac{\text{Sin}^2\text{B Cos}^4\text{A}+\text{Cos}^2\text{B}}{\text{Cos}^2\text{B Sin}^2\text{B}}}$
12. $=\dfrac{\sqrt{\text{Cos}^2\text{A Sin}^2\text{A Cos}^2\text{B Sin}^2\text{B}+\text{Sin}^2\text{B Cos}^4\text{A}+\text{Cos}^2\text{B}}}{\text{Cos B Sin B}}$
13. Let $P=\sqrt{\text{Cos}^2\text{A Sin}^2\text{A Cos}^2\text{B Sin}^2\text{B}+\text{Sin}^2\text{B Cos}^4\text{A}+\text{Cos}^2\text{B}}$
14. Then $h=\dfrac{P}{\text{Cos B Sin B}}$
15. $\text{Sin G}_4=\dfrac{c}{h}$
16. $=\dfrac{\text{Cos A Sin A Cos B Sin B}}{P}$
17. $\text{Sin G}_2=\dfrac{e}{f}$
18. $=\dfrac{\dfrac{\text{Cos}^2\text{A}}{\text{Cos B}}}{\dfrac{M}{\text{Cos B Sin B}}}$
19. $=\dfrac{\text{Cos}^2\text{A Sin B}}{M}$
20. $\text{Cos G}_2=\dfrac{d}{f}$
21. $=\dfrac{\dfrac{1}{\text{Sin B}}}{\dfrac{M}{\text{Cos B Sin B}}}$
22. $=\dfrac{\text{Cos B}}{M}$
23. $g=d\text{ Cos G}_2$

ANALYTIC PROOFS

24. $= \left(\frac{1}{\text{Sin B}}\right)\left(\frac{\text{Cos B}}{\text{M}}\right)$

25. $= \frac{\text{Cos B}}{\text{M Sin B}}$

26. $\text{m} = \text{g Sin G}_4$

27. $= \left(\frac{\text{Cos B}}{\text{M Sin B}}\right)\left(\frac{\text{Cos A Sin A Cos B Sin B}}{\text{P}}\right)$

28. $= \frac{\text{Cos A Sin A Cos}^2\text{B}}{\text{M P}}$

29. $\text{k} = \text{d Sin G}_2$

30. $= \left(\frac{1}{\text{Sin B}}\right)\left(\frac{\text{Cos}^2\text{A Sin B}}{\text{M}}\right)$

31. $= \frac{\text{Cos}^2\text{A}}{\text{M}}$

32. $\text{s} = \sqrt{\text{k}^2 + \text{m}^2}$

33. $= \sqrt{\left(\frac{\text{Cos}^2\text{A}}{\text{M}}\right)^2 + \left(\frac{\text{Cos A Sin A Cos}^2\text{B}}{\text{M P}}\right)^2}$

34. $= \frac{\text{Cos A}}{\text{M P}} \sqrt{\text{P}^2\text{Cos}^2\text{A} + \text{Cos}^4\text{B Sin}^2\text{A}}$

35. Let $\text{N} = \sqrt{\text{P}^2\text{Cos}^2\text{A} + \text{Cos}^4\text{B Sin}^2\text{A}}$ (for convenience)

36. Then $\text{s} = \frac{\text{N Cos A}}{\text{M P}}$

37. $\text{Sin G}_3 = \frac{\text{m}}{\text{s}}$

38. $= \dfrac{\frac{\text{Cos A Sin A Cos}^2\text{B}}{\text{M P}}}{\frac{\text{N Cos A}}{\text{M P}}}$

39. $= \frac{\text{Sin A Cos}^2\text{B}}{\text{N}}$

40. $\text{Cos G}_3 = \frac{\text{k}}{\text{s}}$

41. $= \dfrac{\frac{\text{Cos}^2\text{A}}{\text{M}}}{\frac{\text{N Cos A}}{\text{M P}}}$

42. $= \frac{\text{P Cos A}}{\text{N}}$

43. $\text{n} = \text{g Tan}(\text{B}-\text{G}_2)$

44. $= \left(\frac{\text{Cos B}}{\text{M Sin B}}\right) \text{Tan}(\text{B}-\text{G}_2)$

45. $= \frac{\text{Cos B Sin}(\text{B}-\text{G}_2)}{\text{M Sin B Cos}(\text{B}-\text{G}_2)}$

46. $= \frac{\text{Cos B}(\text{Sin B Cos G}_2 - \text{Cos B Sin G}_2)}{\text{M Sin B}(\text{Cos B Cos G}_2 + \text{Sin B Sin G}_2)}$

ANALYTIC PROOFS

47. $$=\frac{\text{Cos B}\left(\frac{\text{Sin B Cos B}}{\text{M}}-\frac{\text{Cos B Cos}^2\text{A Sin B}}{\text{M}}\right)}{\text{M Sin B}\left(\frac{\text{Cos}^2\text{B}}{\text{M}}+\frac{\text{Cos}^2\text{A Sin}^2\text{B}}{\text{M}}\right)}$$

48. $$=\frac{\text{Cos B (Sin B Cos B}-\text{Cos B Cos}^2\text{A Sin B)}}{\text{M Sin B (Cos}^2\text{B}+\text{Cos}^2\text{A Sin}^2\text{B)}}$$

49. $$=\frac{\text{Cos B (Cos B}-\text{Cos B Cos}^2\text{A)}}{\text{M (Cos}^2\text{B}+\text{Cos}^2\text{A Sin}^2\text{B)}}$$

50. $$=\frac{\text{Cos}^2\text{B (1}-\text{Cos}^2\text{A)}}{\text{M (Cos}^2\text{B}+\text{Cos}^2\text{A Sin}^2\text{B)}}$$

51. $$=\frac{\text{Cos}^2\text{B Sin}^2\text{A}}{\text{M (Cos}^2\text{B}+\text{Cos}^2\text{A Sin}^2\text{B)}}$$

52. $p=m \text{ Tan } G_3$

53. $t=n-p$

54. $=n-m \text{ Tan } G_3$

55. $y=t \text{ Cos } G_3$

56. $=(n-m \text{ Tan } G_3) \text{ Cos } G_3$

57. $v=\frac{m}{\text{Cos } G_3}$

58. $w=t \text{ Sin } G_3$

59. $=(n-m \text{ Tan } G_3) \text{ Sin } G_3$

60. $x=v+w$

61. $=\frac{m}{\text{Cos } G_3}+(n-m \text{ Tan } G_3) \text{ Sin } G_3$

Statement for reduction

62. $\text{Tan } \angle 6=\frac{y}{x}$

63. $$=\frac{(n-m \text{ Tan } G_3)(\text{Cos } G_3)}{\frac{m}{\text{Cos } G_3}+(n-m \text{ Tan } G_3) \text{ Sin } G_3}$$

64. $$=\frac{(n-m \text{ Tan } G_3) \text{ Cos}^2 G_3}{m+(n-m \text{ Tan } G_3) \text{ Sin } G_3 \text{ Cos } G_3}$$

65. $$=\frac{\left(n-m\frac{\text{Sin } G_3}{\text{Cos } G_3}\right) \text{Cos}^2 G_3}{m+\left(n-m\frac{\text{Sin } G_3}{\text{Cos } G_3}\right) \text{Sin } G_3 \text{ Cos } G_3}$$

66. $$=\frac{(n \text{ Cos } G_3-m \text{ Sin } G_3) \text{ Cos } G_3}{m+(n \text{ Cos } G_3-m \text{ Sin } G_3) \text{ Sin } G_3}$$

67. $$=\frac{n \text{ Cos}^2 G_3-m \text{ Sin } G_3 \text{ Cos } G_3}{m+n \text{ Cos } G_3 \text{ Sin } G_3-m \text{ Sin}^2 G_3}$$

68. $$=\frac{n \text{ Cos}^2 G_3-m \text{ Sin } G_3 \text{ Cos } G_3}{m(1-\text{Sin}^2 G_3)+n \text{ Cos } G_3 \text{ Sin } G_3}$$

69. $$=\frac{n \text{ Cos}^2 G_3-m \text{ Sin } G_3 \text{ Cos } G_3}{m \text{ Cos}^2 G_3+n \text{ Cos } G_3 \text{ Sin } G_3}$$

ANALYTIC PROOFS

70. $$=\frac{n\ \mathrm{Cos}\ G_3 - m\ \mathrm{Sin}\ G_3}{m\ \mathrm{Cos}\ G_3 + n\ \mathrm{Sin}\ G_3}$$

Hence by substitution

71. $$\mathrm{Tan}\ \angle 6=\frac{\left(\frac{\mathrm{Cos}^2B\ \mathrm{Sin}^2A}{M(\mathrm{Cos}^2B+\mathrm{Cos}^2A\,\mathrm{Sin}^2B)}\right)\left(\frac{P\ \mathrm{Cos}\ A}{N}\right)-\left(\frac{\mathrm{Cos}\ A\ \mathrm{Sin}\ A\ \mathrm{Cos}^2B}{M\ P}\right)\left(\frac{\mathrm{Sin}\ A\ \mathrm{Cos}^2B}{N}\right)}{\left(\frac{\mathrm{Cos}\ A\ \mathrm{Sin}\ A\ \mathrm{Cos}^2B}{M\ P}\right)\left(\frac{P\ \mathrm{Cos}\ A}{N}\right)+\left(\frac{\mathrm{Cos}^2B\ \mathrm{Sin}^2A}{M(\mathrm{Cos}^2B+\mathrm{Cos}^2A\ \mathrm{Sin}^2B)}\right)\left(\frac{\mathrm{Sin}\ A\ \mathrm{Cos}^2B}{N}\right)}$$

72. $$=\frac{\frac{P\ \mathrm{Cos}^2B\ \mathrm{Sin}^2A\ \mathrm{Cos}\ A}{MN(\mathrm{Cos}^2B+\mathrm{Cos}^2A\ \mathrm{Sin}^2B)}-\frac{\mathrm{Cos}\ A\ \mathrm{Sin}^2A\ \mathrm{Cos}^4B}{M\ P\ N}}{\frac{P\ \mathrm{Cos}^2A\ \mathrm{Sin}\ A\ \mathrm{Cos}^2B}{M\ P\ N}+\frac{\mathrm{Cos}^4B\ \mathrm{Sin}^8A}{MN(\mathrm{Cos}^2B+\mathrm{Cos}^2A\ \mathrm{Sin}^2B)}}$$

73. $$=\frac{P^2\mathrm{Cos}\ A\ \mathrm{Cos}^2B\ \mathrm{Sin}^2A-\mathrm{Cos}^4B\ \mathrm{Sin}^2A\ \mathrm{Cos}\ A\ (\mathrm{Cos}^2B+\mathrm{Cos}^2A\ \mathrm{Sin}^2B)}{P\ \mathrm{Cos}^2A\ \mathrm{Cos}^2B\ \mathrm{Sin}\ A\,(\mathrm{Cos}^2B+\mathrm{Cos}^2A\ \mathrm{Sin}^2B)+P\ \mathrm{Cos}^4B\ \mathrm{Sin}^3A}$$

74. $$=\frac{\mathrm{Sin}^2A\ \mathrm{Cos}^2B\ \mathrm{Cos}\ A\ [P^2-\mathrm{Cos}^2B\,(\mathrm{Cos}^2B+\mathrm{Cos}^2A\ \mathrm{Sin}^2B}{P\ \mathrm{Sin}\ A\ \mathrm{Cos}^2B\ \ [\mathrm{Cos}^2A\,(\mathrm{Cos}^2B+\mathrm{Cos}^2A\ \mathrm{Sin}^2B)\ +\ \mathrm{Cos}\ B\ \mathrm{Si}}$$

75. $$=\frac{\mathrm{Sin}\ A\ \ \mathrm{Cos}\ A\ [P^2-\mathrm{Cos}^2B\ (\mathrm{Cos}^2B+\mathrm{Cos}^2A\ \mathrm{Sin}^2B)]}{P\ [\mathrm{Cos}^2A\ \mathrm{Cos}^2B+\mathrm{Cos}^4A\ \mathrm{Sin}\ B+\mathrm{Cos}^2B\ \mathrm{Sin}^2A]}$$

76. $$=\frac{\mathrm{Sin}\ A\ \mathrm{Cos}\ A\ [P^2-\mathrm{Cos}^2B\ (\mathrm{Cos}^2B+\mathrm{Cos}^2A\ \mathrm{Sin}^2B)]}{P\ [\mathrm{Cos}^2B\ (\mathrm{Cos}^2A+\mathrm{Sin}^2A)+\mathrm{Cos}^4A\ \ \mathrm{Sin}^2B]}$$

77. $$=\frac{\mathrm{Sin}\ A\ \ \mathrm{Cos}\ A\ [P^2-\mathrm{Cos}^2B(\mathrm{Cos}^2B+\mathrm{Cos}^2A\ \ \mathrm{Sin}^2B)]}{P\ [\mathrm{Cos}^2B+\mathrm{Cos}^4A\ \mathrm{Sin}^2B]}$$

78. $$\mathrm{Cos}\ \angle 4=\frac{d}{h}$$

79. $$=\frac{\frac{1}{\mathrm{Sin}\ B}}{\frac{P}{\mathrm{Sin}\ B\ \mathrm{Cos}\ B}}$$

80. $$=\frac{\mathrm{Cos}\ B}{P}$$

81. $\therefore P\ \mathrm{Cos}\ \angle 4=\mathrm{Cos}\ B$

82. $$P=\frac{\mathrm{Cos}\ B}{\mathrm{Cos}\ \angle 4}$$

83. $$P^2=\mathrm{Cos}^2A\ \mathrm{Sin}^2A\ \ \mathrm{Cos}^2B\ \ \mathrm{Sin}^2B+\mathrm{Sin}^2B\ \mathrm{Cos}^4A+\mathrm{Cos}^2B$$

84. $$\mathrm{Tan}\angle 6=\frac{\mathrm{Sin}A\mathrm{Cos}\ A\mathrm{Cos}\angle 4[\mathrm{Cos}^2A\,\mathrm{Sin}^2A\,\mathrm{Cos}^2B\,\mathrm{Sin}^2B+\mathrm{Sin}^2B\,\mathrm{Cos}^4A+\mathrm{Cos}^2B-\mathrm{Cos}^4B-\mathrm{Cos}^2B\mathrm{Sin}^2B\ \mathrm{Cos}^2A]}{\mathrm{Cos}\ B\ [\mathrm{Cos}^2B+\mathrm{Cos}^4A\ \mathrm{Sin}^2B]}$$

85. $$=\frac{\mathrm{Sin}\ A\ \mathrm{Cos}\ \ A\ \mathrm{Cos}\ \angle 4\ [\mathrm{Sin}^2B\ \mathrm{Cos}^2B\ \mathrm{Cos}^2A\ (\mathrm{Sin}^2A-1)+\mathrm{Cos}^2B\ \ (1-\mathrm{Cos}^2B)+\mathrm{Sin}^2B\ \mathrm{Cos}^4A]}{\mathrm{Cos}\ B\ [\mathrm{Cos}^2B+\mathrm{Cos}^4A\ \mathrm{Sin}^2B]}$$

86. $$=\frac{\mathrm{Sin}\ A\ \mathrm{Cos}\ \ A\ \ \mathrm{Cos}\ \angle 4\ \ [-\mathrm{Cos}^4A\ \mathrm{Sin}^2B\ \mathrm{Cos}^2B+\mathrm{Cos}^2B\ \mathrm{Sin}^2B+\mathrm{Sin}^2B\ \mathrm{Cos}^4A]}{\mathrm{Cos}\ B\ \ [\mathrm{Cos}^2B+\mathrm{Cos}^4A\ \mathrm{Sin}^2B]}$$

87. $$=\frac{\mathrm{Sin}\ A\ \mathrm{Cos}\ A\ \ \mathrm{Cos}\ \angle 4\ [\mathrm{Cos}^4A\ \mathrm{Sin}^2B\ \ (1-\mathrm{Cos}^2B)+\mathrm{Cos}^2B\ \mathrm{Sin}^2B]}{\mathrm{Cos}\ B\ (\mathrm{Cos}^2B+\mathrm{Cos}^4A\ \mathrm{Sin}^2B)}$$

88. $$=\frac{\mathrm{Sin}\ A\ \ \mathrm{Cos}\ \ A\ \mathrm{Cos}\ \angle 4(\mathrm{Cos}^4A\ \mathrm{Sin}^4B+\mathrm{Cos}^2B\ \mathrm{Sin}^2B)}{\mathrm{Cos}\ B\ (\mathrm{Cos}^2B+\mathrm{Cos}^4A\ \mathrm{Sin}^2B)}$$

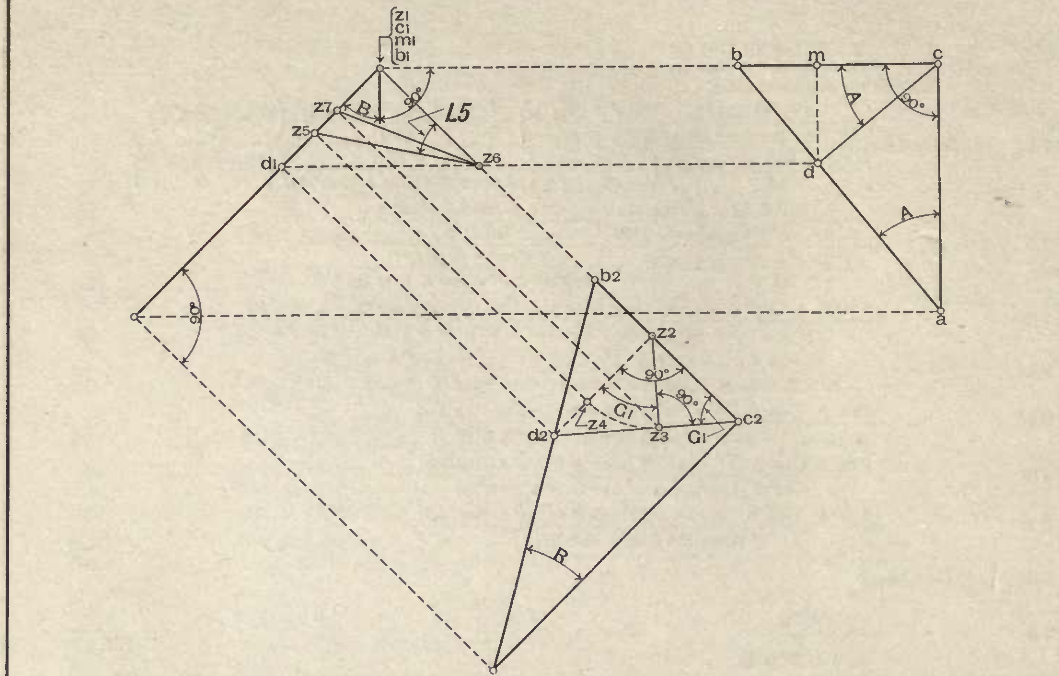
89. $$=\frac{\mathrm{Sin}\ A\ \mathrm{Cos}\ \ A\ \mathrm{Cos}\ \angle 4\ \mathrm{Sin}^2B\ (\mathrm{Cos}^4A\ \mathrm{Sin}^2B+\mathrm{Cos}^2B)}{\mathrm{Cos}\ B\ (\mathrm{Cos}^2B+\mathrm{Cos}^4A\ \mathrm{Sin}^2B)}$$

90. $$=\frac{\mathrm{Sin}\ A\ \mathrm{Cos}\ \ A\ \ \mathrm{Cos}\ \angle 4\ \mathrm{Sin}^2B}{\mathrm{Cos}\ B}$$

91. But, $\mathrm{Tan}\ \angle 3=\mathrm{Sin}\ A\ \mathrm{Cos}\ A\ \mathrm{Sin}\ B\ \mathrm{Tan}\ B$

92. $$=\frac{\mathrm{Sin}\ A\ \mathrm{Cos}\ A\ \mathrm{Sin}^2B}{\mathrm{Cos}\ B}$$

93. $\therefore\ \mathrm{Tan}\ \angle 6=\mathrm{Cos}\ \angle 4\ \mathrm{Tan}\ \angle 3$

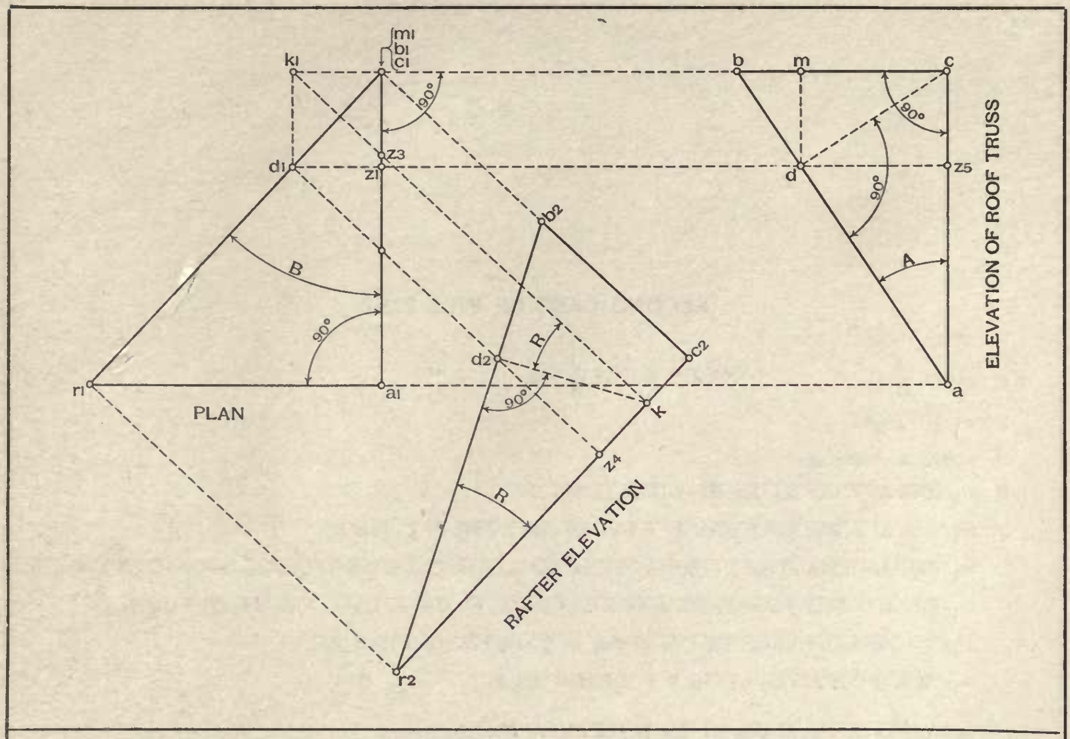


Tangent $\angle 5$

Draw $d_2, z_2 \perp b_2, c_2$
Pass a plane thru $z_2 \perp d_2, c_2$
This plane seen in plan view appears as surface z_2, z_6, z_7
Revolve this plane about z_2, z_6 to z_4
This plane then seen in plan view appears as surface z_2, z_6, z_5

1. $\text{Tan } \angle 5 = \dfrac{z_1, z_5}{z_1, z_6}$
2. $z_1, z_5 = z_2, z_8$
3. $= (z_2, d_2) \text{ Cos } G_1$
4. $= (m_1, d_1) \text{ Cos } G_1$
5. $= (d, m) \text{ Sec B Cos } G_1$
6. $= \text{Sin}^2\text{A Sec B Cos } G_1$
7. $z_1, z_6 = (d, m) \text{ Csc B}$
8. $= \text{Sin}^2\text{A Csc B}$
9. $\therefore \text{Tan } \angle 5 = \dfrac{\text{Sin}^2\text{A Sec B Cos } G_1}{\text{Sin}^2\text{A Csc B}}$
10. $= \dfrac{\text{Sin B Cos } G_1}{\text{Cos B}}$
11. $= \text{Tan B Cos } G_1$
12. $\text{But Cos } G_1 = \dfrac{c_2, z_2}{c_2, d_2}$
13. $= \dfrac{m, c}{f_1, c_1}$ (See Page 10)
14. $= \dfrac{\text{Sin A Cos A}}{f, c \text{ Sec } \angle 1}$
15. $= \dfrac{\text{Sin A Cos A}}{\text{Sin A Sec } \angle 1}$
16. $= \text{Cos A Cos } \angle 1$
17. Hence $\text{Tan } \angle 5 = \text{Tan B Cos A Cos } \angle 1$
18. $\text{But Cos } \angle 3 = \dfrac{c_2, z_8}{c_2, d_2}$
19. $c_2, z_8 = c_2, r_2 \text{ Sin R}$
20. $= \dfrac{\text{Sin R}}{\text{Cos B}}$
21. $c_2, d_2 = \text{Sin A Sec } \angle 1$
22. $\therefore \text{Cos } \angle 3 = \dfrac{\dfrac{\text{Sin R}}{\text{Cos B}}}{\dfrac{\text{Sin A}}{\text{Cos } \angle 1}}$
23. $= \dfrac{\text{Sin R Cos } \angle 1}{\text{Sin A Cos B}}$
24. $\text{And Tan } \angle 4 = \text{Cos}^2\text{A Tan B Sec R}$
25. $\text{Cos } \angle 3 \text{ Tan } \angle 4 = \left(\dfrac{\text{Sin R Cos } \angle 1}{\text{Sin A Cos B}}\right)\left(\dfrac{\text{Cos}^2\text{A Tan B}}{\text{Cos R}}\right)$
26. $= \dfrac{\text{Tan R Cos } \angle 1 \text{ Cos}^2\text{A Tan B}}{\text{Sin A Cos B}}$
27. $= \dfrac{\text{Tan A Cos B Cos } \angle 1 \text{ Cos}^2\text{A Tan B}}{\text{Sin A Cos B}}$
28. $= \dfrac{\text{Sin A Cos B Cos } \angle 1 \text{ Cos}^2\text{A Tan B}}{\text{Cos A Sin A Cos B}}$
29. $= \text{Cos A Tan B Cos } \angle 1$
30. $\therefore \text{Tan } \angle 5 = \text{Cos } \angle 3 \text{ Tan } \angle 4$

PROOF FOR THE 90° USED IN STYLES C AND D



EXPLANATION FOR 90° BEND LINE ON STYLES C AND D.

Purlin Web Plane seen from Elevation of Main Roof is Line cd.
" " " " in Plan View is Inclined Surface d_1, z_1, c_1, k_1.
Rafter Lug Plane seen in Rafter Elevation is Line d_2, k.
" " " " " Plan View is Inclined Surface d_1, z_3, k_1.
These Two Planes produced intersect on Line d_1, k_1.
Hence if k_1, c_1 equals in length d_1, z_1, then will angle k_1, d_1, z_1 be 90° in all cases.

STATEMENT OF VALUES.

ac=Unity
$cd=\sin A$
$ad=\cos A$
$z_1, a_1=\cos^2 A$
$z_1, m_1=\sin^2 A$
$d_1, m_1=\sin^2 A \sec B$
$z_1, d_1=\sin^2 A \tan B$
$d_1, r_1, r_2, z_4=\cos^2 A \sec B$
$d_2, r_2=\cos^2 A \sec B \sec R$
$d_2, z_4=d, z_5=\cos A \sin A$
$z_4, k=\cos A \sin A \tan R$
$d_2, b_2=\sin^2 A \sec B \sec R$
$k, c_2=\sin^2 A \sec B-\cos A \sin A \tan R$
$k_1, c_1=(k, c_2) \csc B$

$=[\sin^2 A \sec B-\cos A \sin A \tan R] \csc B$

PROOF.

1. $(\sin^2 A \sec B-\cos A \sin A \tan R) \csc B=\sin^2 A \tan B$
2. $\dfrac{\dfrac{\sin^2 A}{\cos B}-\cos A \sin A \tan R}{\sin B}=\sin^2 A \tan B$
3. $\dfrac{\sin^2 A-\cos A \sin A \cos B \tan R}{\cos B \sin B}=\sin^2 A \tan B$
4. $\sin^2 A-\cos A \sin A \cos B \tan R=\sin^2 A \tan B \cos B \sin B$
5. $\sin^2 A-\cos A \sin A \cos B \tan R=\sin^2 A \sin^2 B$
6. $\tan R=\tan A \cos B$.
7. $\sin^2 A-\cos A \sin A \cos B \tan A \cos B=\sin^2 A \sin^2 B$.
8. $\sin^2 A-\dfrac{\cos A \sin A \cos^2 B \sin A}{\cos A}=\sin^2 A \sin^2 B$
9. $\sin^2 A-\sin^2 A \cos^2 B=\sin^2 A \sin^2 B$
10. $1-\cos^2 B=\sin^2 B$
11. $1=\sin^2 B+\cos^2 B$ or $1=1$

Hence Eq. 11 being true, proves Eq. 1 to be true.

SECOND CASE OF PIPE LINE

$\frac{1}{2} F = \cos\frac{X}{2}$ $\qquad \cos X = 2\cos^2\frac{X}{2} - 1$

$F = \sqrt{C^2 + E^2}$

$C = \sin A1 + \sin A_2$

$E = \sqrt{(\cos A1 + \cos A_2 \cos B)^2 + (\cos A_2 \sin B)^2}$

$F = \sqrt{(\sin A1 + \sin A_2)^2 + (\cos A1 + \cos A_2 \cos B)^2 + (\cos A_2 \sin B)^2}$

$= \sqrt{(\sin^2 A1 + 2\sin A1 \sin A_2 + \sin^2 A_2) + (\cos^2 A1 + 2\cos A1 \cos A_2 \cos B + \cos^2 A_2 \cos^2 B) + (\cos^2 A_2 \sin^2 B)}$

$= \sqrt{\sin^2 A1 + \cos^2 A1 + \cos^2 A_2(\sin^2 B + \cos^2 B) + \sin^2 A_2 + 2\cos A1 \cos A_2 \cos B + 2\sin A1 \sin A_2}$

$= \sqrt{1 + \cos^2 A_2(1) + \sin^2 A_2 + 2\cos A1 \cos A_2 \cos B + 2\sin A1 \sin A_2}$

$= \sqrt{2 + 2\cos A1 \cos A_2 \cos B + 2\sin A1 \sin A_2}$

$\cos\frac{X}{2} = \frac{1}{2}\sqrt{2 + 2\cos A1 \cos A_2 \cos B + 2\sin A1 \sin A_2}$

$\cos X = \frac{2(2 + 2\cos A1 \cos A_2 \cos B + 2\sin A1 \sin A_2)}{4} - 1$

$= \cos A1 \cos A_2 \cos B + \sin A1 \sin A_2$

When
A1 or $A_2 = 0$
above formula becomes
$\cos X = \cos A \cos B$
which is same as first case

The following formula (proof omitted) was developed by Mr. C. W. L. Filkins.

$\cos X = \frac{\sin A_2 \cos C_2}{\sin C_1}$